Multi-user generic channel equalization and sources separation for MIMO communication systems

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***Abstract*—In this work, we study blind equalization techniques to mitigate inter-symbol interference (ISI), and blind sources separation (BSS) in multiple inputs multiple outputs (MIMO) communication systems. Mainly, we are interested in the multi-user generic blind equalizer (MU-GBE). A MU-GBE has no prior information about the transmission channels and the used constellations. To solve this challenge, a joint MU-GBE, based on a new multi-criteria cost function and automatic modulation classification (AMC) is proposed. The new multi-users multi-criteria cost function is based on the probability density fitting (PDF) and the K-nearest neighbors (KNN) algorithm is considered for the AMC stage. This approach is implemented in its linear and nonlinear versions. For the nonlinear case, we use a neural network. Numerical results, in terms of mean square error (MSE) and symbol error rate (SER) with quadrature amplitude modulation (QAM) signals show that our multi users multi-criteria GBE is effective in alleviating the ISI.**

***Index Terms*—Blind equalization, Blind source separation, probability density fitting, MIMO communication system, neural network, automatic modulation classification.**

I. Introduction

Blind equalization and blind sources separation in a MIMO communication system are hot research topics, especially in the Internet of Things (IoT) communication system [1] [2], targeting the development of efficient and low-complexity algorithms that reduce ISI and blindly separate sources. This will avoid the waste of bandwidth resulting from the training data. To estimate the transmitted signal, the MU-GBE blindly mitigates the ISI and separates the sources. Then, it robustly classifies the transmitted modulation using a suitable AMC technique. Massive access is of great interest in the Internet of Things (IoT) communication system, so we opted to provide our contribution with a MIMO communication system model.

In the literature, some works address generic equalization. We can cite the multi-branch architecture [3], it uses several branches each linked to a constellation and the one that gives the smallest error matches the transmitted constellation. Other algorithms use the cost function relative to the quadrature phase shift keying (QPSK) constellation to equalize any transmit constellation belonging to the phase-shift keying modulation (PSK) or the QAM modulation [4] [5]. As a result, the output of the equalizer is constituted of symbols belonging to the transmitted constellation and compressed in the unit radius. All these previously cited works address the generic blind equalization and apply AMC after the equalization step. In [6], authors propose a multi-criteria generic channel equalizer and they consider an updated PDF based criteria, corresponding to many assumed possible constellations, that are used jointly with an automatic modulation classification based on KNN algorithm.

In this paper, we proposes a new generic equalizer for MIMO communication systems that combines jointly blind equalization and AMC. To the best of our knowledge, this is the first time that a generic blind equalizer (GBE) is proposed with a MIMO communication system and where are simultaneously applied blind equalization and AMC.

Blind equalizers based on PDF like the stochastic quadratic distance (SQD) and multi modulus SQD (MSQD-ℓ_p) outperform equalizers based on high order statistical properties (HOSP) like constant modulus algorithm (CMA) and multi modulus algorithm (MMA) [7]. In [5] and for SISO communication systems, generic equalizer MSQD-$\ell_{2_{gen}}$ based on MSQD-ℓ_2 criterion outperforms generic equalizer CMA$_{gen}$ based on CMA. Therfore, in [6] and for SISO communication systems, the multi-criteria generic equalizer MC-MSQD-$\ell_{2_{gen}}$ outeprforms the MSQD-$\ell_{2_{gen}}$. For this reason, we have considered MSQD-ℓ_2 criterion to propose a new multi users multi-criteria generic cost function. In addition, we implemented the proposed new MU-GBE in linear and nonlinear cases. The nonlinear case banks complex-valued neural networks (CV-NN).

For the AMC step, in the literature, various algorithms for modulation classification are considered such us Maximum-likelihood [8] [9] and order cyclic cumulants [10] [11]. In this paper, we use the KNN algorithm with the fourth-order cumulants [12] [13].

Consequently, the main contributions of our work are summarized as follows:

- Derivation of a new multi-criteria cost function for a multi-users generic equalization.
- Multi-user generic blind equalization and sources separation.

II. Signal and system model

MIMO system model with linear time-invariant MIMO channel is illustrated in Fig.1, K and Q are the number of transmit antennas and receive antennas respectively.

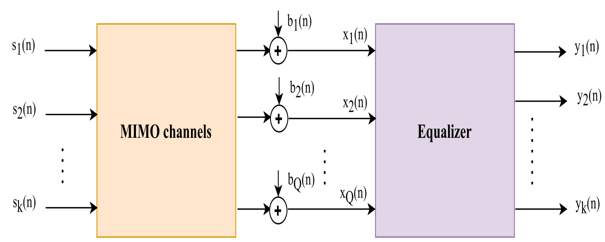

Fig. 1: Based MIMO system model.

In the sequel, we consider the following parameters and notations. We note that lowercase letters, e.g. x and bold lowercase letters e.g. $\boldsymbol{x}$, stand for scalars and vectors respectively. $\{s_i(n), i = 1, ..., k\}$ are the input signals at a given time n, $\{h_{ij}(n), i = 1, ..., Q, j = 1, ..., K\}$ are the channel impulse responses, $\{b_i(n), i = 1, ..., Q\}$ are the additive white Gaussian noise at a given time n, $\{x_i(n), i = 1, ..., Q\}$ denote the received signals at a given time n. Assuming that the length of each channel h_{ij} is L_h we have :

$$x_i(n) = \sum_{j=1}^{K} \mathbf{h}_{ij}^T \mathbf{s}_j(n) + b_i(n) i = 1...Q, \quad (1)$$

where $\mathbf{s}_j(n) = [s_j(n), s_j(n-1), ..., s_j(n-L_{h-1})]$, and $\mathbf{h}_{ij} = [h_{ij}(0), h_{ij}(1), ..., h_{ij}(L_{h-1})]$.

We denote by G the equalizer function such as :

- **Linear context :** $G = \{w_{ij}, i = 1, ..., K, j = 1, ..., Q\}$ is the equalizer impulse response and $\{y_i(n), i = 1, ..., k\}$ are the equalized signals at time n. Assume that the length of each equalizer w_{ij} is L_w, we have :

$$y_i(n) = \sum_{j=1}^{Q} w_{ij}^T x_j(n) i = 1...k, \tag{2}$$

  where
  $\mathbf{x}_j(n) = [x_j(n), x_j(n-1), ..., x_j(n-L_w+1)]$,
  and
  $\mathbf{w}_{ij} = [w_{ij}(0), w_{ij}(1), ..., w_{ij}(L_w-1)]$.
- **Nonlinear context :** G is a neural network that will be detailed in section III.B.

III. Equalizer model

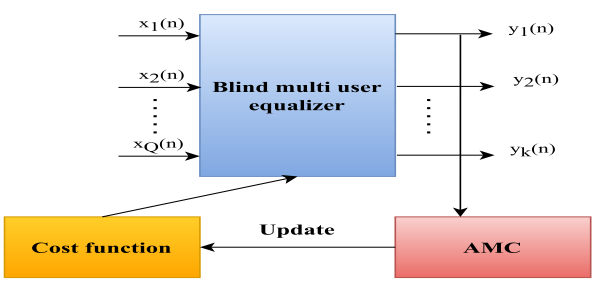

Fig. 2: Multi-user generic equalizer architecture.

In this work, two contributions are proposed. The first one is a generic linear multi users multi-criteria blind equalizer. The second one is a generic nonlinear multi users multi-criteria blind equalizer using a CV-NN. The new multi users multi-criteria cost function for the equalization part and the algorithm used for the AMC for each contribution will be detailed below.

A. Multi-users Multi-criteria MSQD-$\ell_{p_{gen}}$ (MU-MC-MSQD-$\ell_{p_{gen}}$) in linear case

As indicated in the previous section, equalizers based on PDF outperform equalizers with HOSP [7] and the multi criteria generic channel equalizer proposed in [6] outperforms the two generic equalizers suggested in [4] and [5]. In addition, decomposing the equalization criterion into an in-phase component and a quadrature component is more efficient than processing two components together, with respect to a phase shift introduced by the transmission channels. These three reasons lead us to derive a new multi users multi-criteria generic cost function in a MIMO system with K transmitter based on the multi-modulus stochastic quadratic distance MSQD-ℓ_p algorithm. For $p = 2$, the MSQD-ℓ_2 cost function is [6] :

$$\begin{aligned} J_{MSQD-\ell 2}(\boldsymbol{w}) = &-\frac{1}{N_s}\sum_{m=1}^{N_s} K_\sigma(|y_R(n)|^2 - |a_R(m)|^2) \\ &- \frac{1}{N_s}\sum_{m=1}^{N_s} K_\sigma(|y_I(n)|^2 - |a_I(m)|^2), \end{aligned} \tag{3}$$

where N_s is the number of symbols in the constellation, it is equals to 16 for 16-QAM modulation, $y_R(n)$ and $y_I(n)$ are the real and the imaginary parts of the equalizer output at a given time n and which depends on the weighted coefficient w, $a_R(m)$ and $a_I(m)$ are the real and the imaginary parts of the m^{th} $(m = 1 \dots N_s)$ symbol in the used constellation and K_σ is a Gaussian kernel with zero means and a variance σ also denoted kernel width.
The multi-user multi-modulus stochastic quadratic distance MU-MSQD-ℓ_2 cost function is derived from (3) as in [14]:

$$J_{MU-MSQD-\ell 2}(\boldsymbol{w}) = \sum_{i=1}^{k} -\frac{1}{N_s}\sum_{m=1}^{N_s} K_\sigma(|y_{i,R}(n)|^2 - |a_R(m)|^2)$$
$$+ \sum_{i=1}^{k} -\frac{1}{N_s}\sum_{m=1}^{N_s} K_\sigma(|y_{i,I}(n)|^2 - |a_I(m)|^2)$$
$$+ \beta \sum_{i,j=1;i\neq j}^{k} \sum_{\delta=\delta_1}^{\delta_2} |C_{i,j}[\delta]|^2 , \tag{4}$$

where $y_{i,R}(n)$ and $y_{i,I}(n)$ denote the real and imaginary part of the i^{th} equalizer output at a given time n respectively, k is the transmitter antennas number, β is a positive number between 0 and 1, δ_1 and δ_2 are set to values that ensure the equalizer convergence, and the cross-correlation term among i^{th} and j^{th} equalizer outputs is used for blind sources separation and can be defined as in [14]:

$$C_{i,j} = E\left[y_i(n)y_j^*(n-\delta)\right].$$

The new multi-users multi-criteria generic cost function is a sum of several variations of (4) one for each constellation order multiplied by an updated penalty factor :

$$J_{MU-MC-MSQD-\ell 2_{gen}}(\boldsymbol{w}) = \sum_{g=1}^{G} \alpha_g J_{MU-MSQD-\ell 2_g}(\boldsymbol{w}), \tag{5}$$

where G is the number of the considered constellations orders.The penalty factor α_g in (5) is updated in each iteration according to the following expression such that we reach, over the iterations, the transmitted constellation cost function :

$$\alpha_g = sigmoid(class_g), \tag{6}$$

where $class_g$ is the number of times where the g^{th} constellation is successful in the classification result after the classification step.
The equalizer coefficients are updated using the stochastic gradient descendent (SGD) algorithm as:

$$w(n+1) = w(n) - \mu \bigtriangledown_w J_{MU-MC-MSQD-\ell 2_{gen}}(\boldsymbol{w})$$
$$= w(n) - \mu \sum_{g=1}^{G} \alpha_g \bigtriangledown_w J_{MU-MSQD-\ell 2_g}(\boldsymbol{w}), \tag{7}$$

where μ is the step size.
In the sequel, we will focus on the explicit expression of (5). For this reason, we begin by calculating the gradient of (4) which leads to that of (5). on the other hand, in [6] we have the gradient of (3) :

$$\bigtriangledown_w J_{MSQD-\ell 2}(w) = \frac{\partial J_{MSQD-\ell 2}(w)}{\partial w_r} + j\frac{\partial J_{MSQD-\ell 2}(w)}{\partial w_i}$$
$$= \frac{1}{\sqrt{2\pi}N_s\sigma^3}\sum_{m=1}^{N_s}(sign(y_R(n))|y_R(n)|$$
$$(|y_R(n)|^2 - |a_R(m)|^2)$$
$$e^-(|y_R(n)|^2 - |a_R(m)|^2)^2/2\sigma^2$$
$$+ jsign(y_I(n))|y_I(n)|(|y_I(n)|^2 - |a_I(m)|^2)$$
$$e^-(|y_I(n)|^2 - |a_I(m)|^2)^2/2\sigma^2)x^*(n), \tag{8}$$

sign() is the signum function.
The gradient of (4) is the sum of (8) for each equalizer output plus the gradient of correlation term :

$$\bigtriangledown_w J_{MU-MSQD-\ell 2}(w) = \sum_{i=1}^{K} \bigtriangledown_w J_{MSQD-\ell 2_i}(w)$$
$$+ 2\beta \sum_{i,j=1;i\neq j}^{k} \sum_{\delta=\delta_1}^{\delta_2} y_i(n)\,|y_j(n-\delta)|^2 . \tag{9}$$

The gradient of (5) is the sum of (9) for each constellation order multiplied by an updated penalty factor.
In order to update the cost function, each equalized symbol is classified using the KNN algorithm with fourth-order cumulant [12] [13] as features. So that we do jointly the equalization and the AMC. The expressions of the p^{th} order cumulant and moment are respectively as follows :

$$C_{pq} = cum(x^{p-q}(x^*)^q)$$
$$and$$
$$M_{pq} = E[x^{p-q}(x^*)^q],$$

where $E[.]$ is the expectation operator.
In particular, consider :

$$C_{40} = M_{40} - 3M_{20}^2$$
$$C_{42} = M_{42} - |M_{20}|^2 - 2M_{21}^2$$

as they are more suitable for M-QAM modulations [15]. A baseline containing C_{40} and C_{42} values for noisy symbols according to various values of signal-to-noise ratio (SNR) and belonging to $\{16, 32, 64, 128, 256\}$-QAM modulations is prepared.
the classifier follows the following steps :

- Calculate $S_1 = C_{40} + C_{42}$ the sum of the last 1000 equalized symbols.
- Calculate the Euclidean distances between S_1 and each sum in the reference base.
- A set of the k nearest neighbors from the baseline is created.
- Since each neighbor matches a specific constellation order, then we select the order is the most repeated order in the previous set.

Finally, according to the AMC, we update the penalty factors following (6). Then (5) will be updated.

B. Multi users Multi-criteria neural network NNMSQD-$\ell_{p_{gen}}$(MU-MC-NNMSQD-$\ell_{p_{gen}}$)

The used CV-NN combined with a KNN classifier is illustrated in Fig.3. It's trained using the complex back propagation (CBP) algorithm [16].

We assume N_k neurons in the k^{th} layer and we denote by ϕ_j^k and x_j^{k+1} the input and the output of the j^{th} neuron in the k^{th} layer, such that :

$$\phi_j^k = \sum_{i=1}^{N_{k-1}} w_{ij}^k x_i^k + \theta_j^k \text{ and} \tag{10}$$

$$x_j^{k+1} = f^k(\mathrm{Re}(\phi_j^k)) + jf^k(\mathrm{Im}(\phi_j^k)), \tag{11}$$

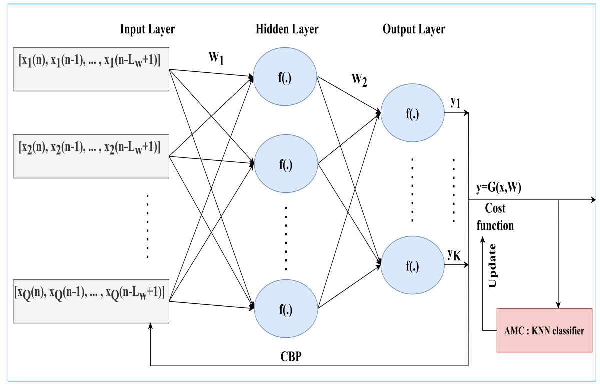

Fig. 3: MIMO CVNN combined with a KNN classifier.

where x_i^k is the k^{th} layer output, w_{ij}^k is the weight between the i^{th} neuron in the k^{th} layer and the j^{th} neuron in the $(k+1)^{th}$ layer, θ_j^k and $f^k(\cdot)$ are the k^{th} layer bias and activation function.
During CV-NN training process, we have two main steps. The first one is forward step where the goal is to calculate the neural network outputs, by applying an activation function on the weighted sum in each layer. The second step is the CBP to update the synaptic weights according to our equalizer criterion. The network includes three layers: input, hidden, and output layers. In this work, we use the same activation function as in [6] :

$$f(x) = x + \alpha * \sin(\pi * x),$$

where π is a mathematical constant approximately equal to 3.14159, α a positive number between 0 and 1 and sin is the trigonometric function. The cost function for the generic multi users multi-criteria neural network equalizer is the same as that of (5) :

$$J_{MU-MC-NNMSQD-\ell_{2_{gen}}}(\boldsymbol{w}) = \sum_{g=1}^{G} \alpha_g J_{MSQD-\ell_{2_g}}. \tag{12}$$

The penalty factor α_g is calculated and updated as in the MU-MC-MSQD-$\ell_{2_{gen}}$ algorithm and for each iteration, we follow the same steps. First, we equalize the received symbol, then we classify it, afterward, we update the neural network weights and finally, we update the penalty factors respectively.

The weights of the neural network are updated using the SGD algorithm as :

$$\begin{aligned} w(n+1) &= w(n) - \mu \nabla_w J_{MU-MC-NNMSQD-\ell_{2_{gen}}}(\boldsymbol{w}) \\ &= w(n) - \mu \sum_{g=1}^{G} \alpha_g \nabla_w J_{MU-MSQD-\ell_{2_g}}(\boldsymbol{w}), \end{aligned} \tag{13}$$

where μ is the step size.

To update the output layer weights, the same process is used such in the linear version, We begin by calculating the gradient of (4) which leads to that of (12). In [6], we have the gradient of (3) for the neural version with SISO communication system and for simplification purposes, we introduce $Q_{R,i}$, $Q_{I,i}$, $\delta^o_{p,i}$, and Υ_i for the i^{th} equalizer output, which are expressed as the following :

$$\begin{aligned} Q_{R,i} &= \frac{1}{N_s\sqrt{2\Pi}\sigma} \sum_{m=1}^{N_s} e^{-\frac{(|y_{R,i}(n)|^2 - |a_R(m)|^2)^2}{2\sigma^2}} \\ &\quad \frac{(|y_{R,i}(n)|^2 - |a_R(m)|^2)}{\sigma^2}, \\ Q_{I,i} &= \frac{1}{N_s\sqrt{2\Pi}\sigma} \sum_{m=1}^{N_s} e^{-\frac{(|y_{I,i}(n)|^2 - |a_I(m)|^2)^2}{2\sigma^2}} \\ &\quad \frac{(|y_{I,i}(n)|^2 - |a_I(m)|^2)}{\sigma^2}, \\ \delta^o_{p,i} &= Q_{R,i} y_{R,i}(n) f^{o'}(\varphi^o_{R,i}) + jQ_{I,i} y_{I,i}(n) f^{o'}(\varphi^o_{I,i}). \end{aligned} \tag{14}$$

$$\Upsilon^o_{p,i} = 2\beta\delta^o_{p,i} \sum_{j=1,j\neq i}^{K} \sum_{\delta=\delta_1}^{\delta_2} |y_j(n-\delta)|^2. \tag{15}$$

The gradient of (4) with respect to output layer weights is calculated as :

$$\nabla_w J_{MU-NNMSQD-\ell 2}(w) = (\delta^o_p + \Upsilon^o_p) I^*_p, \tag{16}$$

where $\varphi^o_{R,i}$, $\varphi^o_{I,i}$, are the i^{th} real and imaginary inputs of the output layer, and I_p is the output of the hidden layer. $f^{o'}(\varphi^o_{R,i})$ and $f^{o'}(\varphi^o_{I,i})$ are the derivatives of the activation function in the output layer applied in the real and imaginary parts of the output layer i^{th} input,

$$\delta^o_p = \begin{pmatrix} \delta^o_{p,1} \\ . \\ . \\ . \\ \delta^o_{p,Q} \end{pmatrix}, \text{ and } \Upsilon^o_p = \begin{pmatrix} \Upsilon^o_{p,1} \\ . \\ . \\ . \\ \Upsilon^o_{p,Q} \end{pmatrix}.$$

The gradient of (12) with respect to the output layer weights is the sum of (16) for each constellation order multiplied by an updated penalty factor.

Also, the gradient of (12) with respect to hidden layer weights is calculated by applying the CBP and SGD algorithms. In [6], we have have the gradient of (3) for neural version with SISO communication system and we simplify the calculation by introducing δ^h_p and Υ^h_p,

$$\begin{aligned} \delta^h_p &= f^{h'}(\varphi^h_R)\Re((\delta^o_p + \Upsilon^o_p) w^{o^*}) \\ &\quad + jf^{h'}(\varphi^h_I)\Im((\delta^o_p + \Upsilon^o_p) w^{o^*}). \end{aligned} \tag{17}$$

$$\Upsilon^h_p = 2\beta\delta^h_p \sum_{j=1,j\neq i}^{K} \sum_{\delta=\delta_1}^{\delta_2} |y_j(n-\delta)|^2. \tag{18}$$

The gradient of (4) with respect to hidden layer weights is calculated as :

$$\nabla_w J_{MU-NNMSQD-\ell 2}(w) = (\delta^h_p + \Upsilon^h_p) X, \tag{19}$$

where φ^h_R, φ^h_I, are the real and imaginary inputs of the hidden layer, and X is the input vector of the equalizer. $f^{h'}(\varphi^h_R)$ and $f^{h'}(\varphi^h_I)$ are the derivatives of the activation function of the hidden layer applied in the real and imaginary parts of the hidden layer input.

The gradient of (12) with respect to hidden layer weights is the sum of (19) for each constellation order multiplied by an updated penalty factor.

Finally, the AMC process is applied using the same principle as for MU-MC-MSQD-$\ell_{2_{gen}}$.

IV. Simulation results

we have simulated our two equalizers: MU-MC-MSQD-$\ell_{2_{gen}}$ and MU-MC-NNMSQD-$\ell_{2_{gen}}$ in the case of MIMO system equipped with two transmitter antennas and four receiver antennas. So we have eight multipath channels, each containing six paths [7]. We obtained the MSE and SER values for various signal-to-noise ratio (SNR) values with a 16-QAM constellation signal.

We have used MU-MSQD-$\ell_{2_{gen}}$ as a linear benchmark and MU-NNMSQD-$\ell_{2_{gen}}$ as a nonlinear neural benchmark. These two benchmarks, consider the cost function relative to the QPSK constellation to equalize any transmitted constellation belonging to the PSK or the QAM modulation.

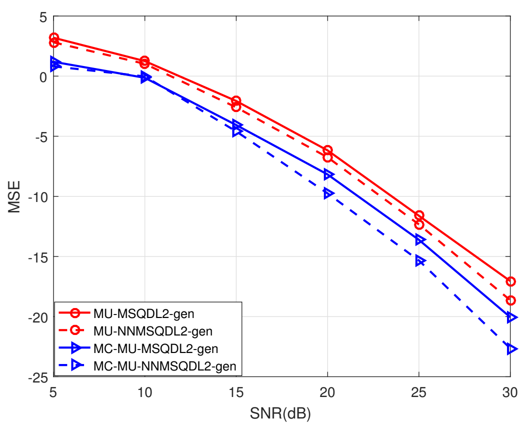

Fig. 4: MSE versus SNR for 16-QAM.

Fig.4 shows that our approach for linear and nonlinear versions outperforms the linear generic equalizer MU-MSQD-$\ell_{2_{gen}}$ and the nonlinear generic equalizer MU-NNMSQD-$\ell_{2_{gen}}$ in terms of MSE. Also, our generic multi-users multi-criteria neural equalizer MC-MU-NNMSQD-$\ell_{2_{gen}}$ outperforms our linear version MU-MC-MSQD-$\ell_{2_{gen}}$.

Fig.5 confirms the MSE results. In particular, for a high SNR values, we notice a better performance where we have an optimal learning phase.

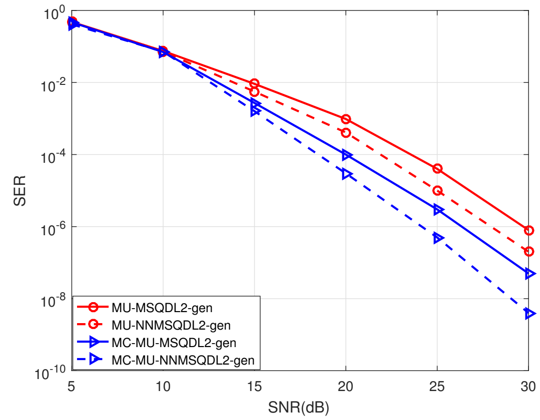

Fig. 5: SER versus SNR for 16-QAM.

V. Conclusion

In conclusion, we can use our approach with new communications techniques, especially in the fifth generation (5G) and beyond which use millimeter waves and power amplifiers (PAs). In this case, we have two challenges to solve, the linear ISI effect resulting from the multi-path channel and the PA non-linearity effects.

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