Dimensioning Methodology for OFDMA Networks

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Extended Abstract *Abstract*—**In this paper, we present a comprehensive methodology to dimension an OFDMA (Orthogonal Frequency Division Multiple Access) network. We divide the dimensioning task into sub-blocks and present implementation details of each block. The first block has the objective of determining the spatial SINR (Signal to Interference plus Noise Ratio) distribution over a cell area and the MCS (Modulation and Coding Scheme) probabilities. This task usually relies on extensive Monte Carlo simulations. In this paper, we propose a semi-analytical approach in order to reduce computational time. The second block evaluates dimensioning parameters based on a markovian approach. Our model takes into account mixed traffic profiles, different scheduling policies and MCS probabilities obtained from the first step. Theses two stages (radio coverage and traffic analysis) form together a complete dimensioning process for OFDMA networks.**

I. INTRODUCTION

In recent years, the demand for high data rate wireless broadband services has increased significantly. Standards like 3GPP LTE (Long Term Evolution) or IEEE 802.16m support key technology enablers for high data rate services including video streaming, internet access and telephony over wireless access medium.

Physical and MAC layers of these standards are characterized by spectrally efficient Multiple Access schemes based on Orthogonal Frequency Division Multiple Access (OFDMA). Accurate and robust performance evaluation methodologies are essential to ensure the optimal utilization of the system features. Additionally, such tools are useful to anticipate the impact of traffic growth on the QoS (Quality of Service) experienced by each subscriber.

Some of the main concerns for QoS are blocking probability for Voice, average and peak data rates for Best-Effort like data traffic. The experienced quality is influenced by the radio channel performance (outage probability, probabilities of MCS: Modulation and Coding Schemes) as well as by the resource allocation strategy.

In this paper, a comprehensive methodology to predict capacity of OFDMA networks is proposed. Simulation based performance analysers exhibit long computation time and high complexity. On the contrary, much more efficient solutions are preferred through the use of analytical models. This framework is then based on analytical modeling. Two different categories of models are described respectively for spectral efficiency, interference prediction and for multi-traffic performance analysis.

Indeed, interference prediction is a major issue in mobile radio networks and has been deeply investigated through system level simulations as in [1]- [2] for IEEE 802.16 but exhibit un-acceptable simulation time for network dimensioning when tens of scenarios have to be tested. Analytical/semi analytical methods have also been proposed in [3]- [4] for CDMA (Code Division Multiple Access) based networks but the analysis that has been carried out for the single carrier case and cannot be directly extended to multi-carrier, frequency diverse OFDMA based networks. In [5], analytical model to predict radio throughput in WiMAX is presented but channel variations (shadowing) are not accounted for. In this paper, we propose a semi-analytical method for the evaluation of SINR (Signal to Interference plus Noise Ratio) spatial distribution and MCS probabilities in a cellular network with different frequency re-use schemes. These MCS probabilities are then used by a Markov chain based traffic analysis.

In the literature, traffic analysis is either based on packet level simulations as in [6] for WiMAX or relies on analytical approaches as in [7]. In this paper, simple traffic related performance modeling based on a markovian approach is proposed. Compared to simulations-based framework, this model enables to instantaneously predict performance metrics. Compared to analytical works, closed-form expressions for all performance metrics are developed. They take into account different scheduling policies and are especially designed to consider the elastic ON/OFF nature of the Internet traffic belonging to the WiMAX Best-Effort service class. This model doesn't only capture the effect of the scheduling policy and of the elastic nature of data traffic but also takes into account the channel variations. Typical performance metrics such as average user throughput, resource utilization can be derived instantaneously from these analytical models as functions of the number of users.

The semi-analytical method used to obtain MCS probabilities and the Markov chain based traffic analysis form together a dimensioning methodology for OFDMA networks.

The paper is organized as follows. In section II, we present the generic dimensioning process that includes the computation of MCS probabilities and the traffic analysis. We detail the semi-analytical method in section III and the traffic analysis in section IV. In section V, we provide an OFDMA network dimensioning example based on the proposed methodology.
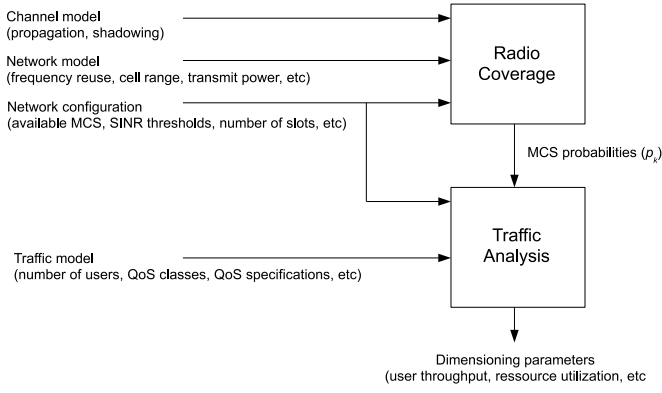
Fig. 1. Network dimensioning process overview.

II. NETWORK DIMENSIONING PROCESS

As shown in Fig. 1, the study of network dimensioning for OFDMA networks (e.g. mobile WiMAX) can be divided into two different components: *Radio Coverage*, which provides MCS spatial probabilities in a cell, and *Traffic Analysis*, which provides dimensioning parameters based on these MCS probabilities. A brief description of these blocks with their respective inputs and outputs is presented hereafter.

A. Radio Coverage

The goal of the *Radio Coverage* block is to provide MCS probabilities for a generic user in a generic cell. These probabilities are derived from a SINR spatial distribution over the cell area and from SINR thresholds, which delimit the different MCS.

The input parameters to this block are: the channel model (this model includes the path-loss model and the shadowing standard deviation), the network model (all parameters related to the network deployment such as the BS (Base Station) transmit power, the frequency reuse scheme, the cell range, the antenna gains, etc) and the network configuration (i.e., parameters related to the deployed technology, e.g., the available MCS, the SINR thresholds, the number of available radio resources per frame, etc). These parameters are mainly based on [8].

From these inputs, we are able to derive the spatial CDF (Cumulative Distribution Function) of $SINR_{eff}$. Here, $SINR_{eff}$ is the effective SINR of all subcarriers of a slot and is computed using physical abstraction models (like Mean Instantaneous Capacity MIC). The CDF is usually obtained through extensive Monte Carlo simulations. As the disadvantage of simulation approach is excessive time consumption, we introduce in section III a semi-analytical method to substitute the simulation approach.

Once the *Radio Coverage* block has furnished the CDF of $SINR_{eff}$, we require thresholds values of different MCS types to calculate MCS probabilities (denoted $(p_k)_{0 \leq k \leq K}$). Considered MCS types, their respective $SINR_{eff}$ threshold values and number of bits per slot (m_k) are given in Tab. I and have been referred from [9]. If $SINR_{eff}$ of a mobile station

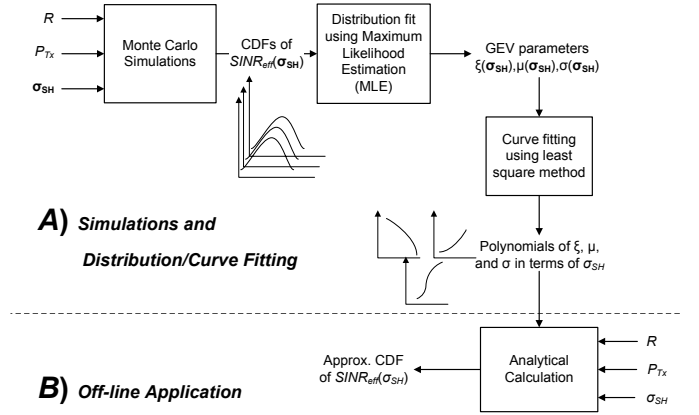| Index k	MCS	bits per slot m_k	$SINR_{eff}$ [dB]
0	Outage	0	< 2.9
1	QPSK 1/2	48	2.9
2	QPSK 3/4	72	6.3
3	16QAM 1/2	96	8.6
4	16QAM 3/4	144	12.7
5	64QAM 2/3	192	16.9
6	64QAM 3/4	216	18

Fig. 2. Overview of proposed semi-analytical method.

(MS) is less than the threshold of the most robust MCS (i.e., less than 2.9 dB), it can neither receive nor transmit anything and is said to be in outage. We call outage as MCS type 0.

B. Traffic Analysis

From the *Traffic Analysis*, we are able to obtain dimensioning parameters such as the average throughput per user, the average radio resource utilization or the average number of simultaneous active users in the cell.

The *Traffic Analysis* block takes as inputs the MCS probabilities, the network configuration and the traffic model. Network configuration parameters consist in N_S, the number of slots in DL sub-frame in a cell (i.e., per three sectors), T_F, the duration of TDD (Time Division Duplex) frame, and the scheduling policy. Traffic model parameters include N, the number of mobiles present in the cell and the different profiles of the traffics generated by those mobiles.

The details of the Markovian approach used to obtain dimensioning parameters are given in section IV.

III. RADIO COVERAGE

The semi-analytical method presented in this paper is based on [10]. A systematic overview of the method is depicted in Fig. 2. The method is divided into two steps: A) Simulations and Distribution/Curve Fitting and B) Off-line Application. The ultimate goal of the method is to obtain the effective SINR distribution and the MCS probabilities without relying on time consuming Monte Carlo simulations. In the following text, the two steps of the method are explained in detail.

Parameter	Value
Reuse type	3x3x3
No. of interfering BS	18 using wraparound technique
Spatial distribution of MS	Uniform random
Number of MS dropped per sector	1
Number of snapshots	10000
Carrier frequency f_c	2.5 GHz
Subcarrier spacing $\triangle f$	10.9375 kHz
TDD frame duration	5 ms
Thermal noise density N_0	-174 dBm/Hz
Shadowing standard deviation σ_{SH}	8.9 dB
Height of BS h_{BS}	32 m
Height of MS h_{MS}	1.5 m
Antenna beam pattern 3GPP2 $G(\psi)$, where ψ is the angle MS subtends with sector boresight	$G_{max} + \max\left[-12\left(\frac{\psi}{\psi_{3dB}}\right)^2, -G_{FB}\right]$
Antenna Gain (boresight) G_{max}	16 dBi
Front-to-back power ratio G_{FB}	25 dB
3-dB beamwidth ψ_{3dB}	70°

A. Simulations and Distribution/Curve Fitting

The generic idea of this step is to run simulations for a specific set of parameters and to approximate the obtained $SINR_{eff}$ spatial distribution by a known distribution, the Generalized Extreme Value (GEV) distribution [11]. We then make the approximation that the GEV parameters are mainly depending on the shadowing standard deviation σ_{SH}. With curve fitting, we are able to express these GEV parameters as polynomials of σ_{SH}. These polynomials will be used for the off-line application.

Spatial distributions of $SINR_{eff}$ are first obtained using Monte Carlo simulations for a given cell range R, a given BS transmission power P_{Tx} and a specified range of shadowing standard deviation σ_{SH} values. The results are specific to a frequency reuse scheme. The parameters (mainly based on [8]) can be found in Tab. II. The details of simulator can be found in [12].

Each distribution of $SINR_{eff}$ is specific to a value of σ_{SH}. With the help of distribution fit (based on maximum likelihood estimation), the GEV distribution parameters (shape parameter ξ, scale parameter σ and location parameter μ), approximating the simulation PDFs (Probability Density Function), are acquired for each value of σ_{SH}.

In order to evaluate the distribution fit, the dissimilarity or error Ξ between GEV and simulation PDFs, φ_{GEV} and φ_{sim}, is quantified as follows [13]:

$$\Xi \triangleq \int_{-\infty}^{\infty} |\varphi_{GEV}(t) - \varphi_{sim}(t)| \, dt. \quad (1)$$

Since the area under a PDF is 1, the maximum value of error can be 2. Hence the value of error can be between 0 and 2 i.e., $0 \leq \Xi \leq 2$.

B. Off-line Application

To calculate $SINR_{eff}$ distribution for any desired value of σ_{SH} in the range specified in section III-A, we no longer require to carry out time consuming Monte Carlo simulations. It is sufficient to find out GEV parameters through polynomials for that value of σ_{SH}.

Then using GEV CDF and thresholds values of $SINR_{eff}$ for different MCS types of Tab. I, probabilities of these MCS can be obtained. We now give some numerical results, and also show that results obtained through this method are applicable for various values of R and P_{Tx}.

C. Numerical Results

In this section, we present the numerical results. The considered frequency reuse type is 3x3x3 (cf. [10] for explanation of frequency reuse types). For Monte Carlo simulations, range of σ_{SH} is considered to be $4, 5, ..., 12$. Other input parameters are $R = 1500$ m and $P_{Tx} = 43$ dBm.

An $SINR_{eff}$ distribution is obtained for each value of σ_{SH}. Using distribution fitting, GEV parameters are determined for each of these distributions. As an example, in Fig. 3, approximation of $SINR_{eff}$ PDF (obtained through simulation) by a GEV PDF for $\sigma_{SH} = 9$ dB is shown. As can be noticed, the two distributions only have a dissimilarity error of 0.052 which is 2.6% of the maximum possible error.
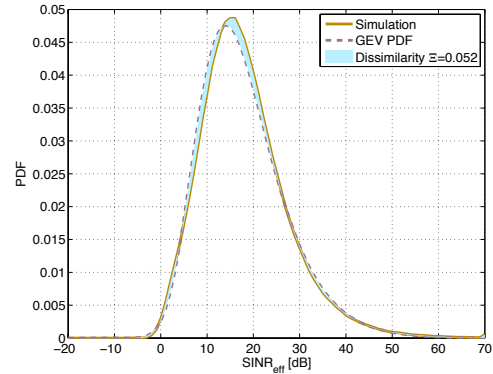

Fig. 3. $SINR_{eff}$ distribution through simulation and GEV polynomial for $\sigma_{SH} = 9$ dB, $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.

GEV parameters, obtained though distribution fitting, are separately plotted against σ_{SH} values in Figs. 4(a), 4(b) and 4(c). With the help of curve fitting, polynomials of the curves approximating these plots are found and are also given in the figures. As can be noted in the figures, the degree of all polynomials never exceeds four. These polynomials can instantaneously give values of GEV parameters for any value of σ_{SH}.

To validate off-line application (cf. section III-B), we choose an arbitrary value $\sigma_{SH} = 7.5$ dB. We calculate the GEV parameters thanks to the polynomials obtained after fitting and get PDF and MCS probabilities. For the same value of σ_{SH} and assuming the values of $R = 1500$ m, $P_{Tx} = 43$ dBm, we find the PDFs and MCS probabilities through simulations. Furthermore, we also check the applicability of results obtained through GEV parameters, with $\sigma_{SH} = 7.5$ dB, for various cell configurations. For this purpose, we fix $\sigma_{SH} = 7.5$ dB and carry out simulations for different values of R and P_{Tx}. The

(a) Shape parameter ξ (b) Scale parameter σ (c) Location parameter μ
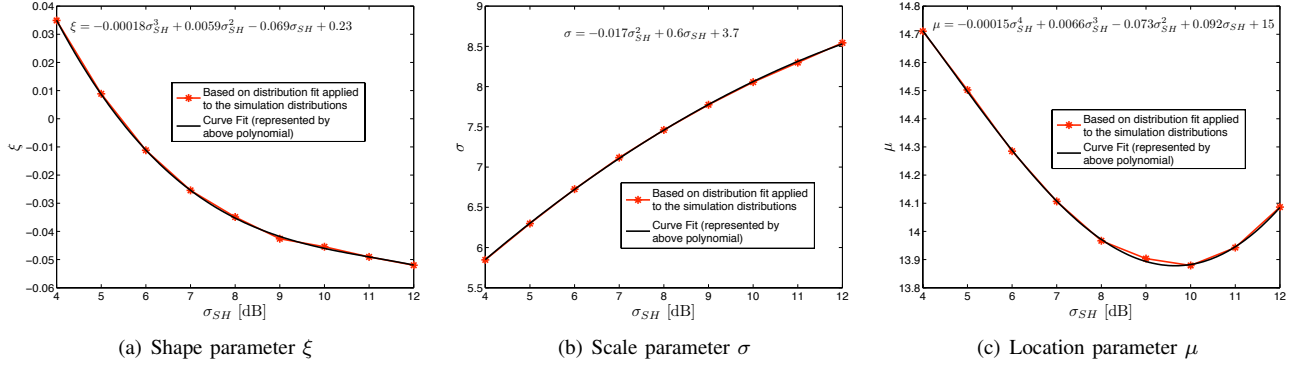
Fig. 4. Shape, scale and location parameters of GEV distribution versus σ_{SH} for $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.

TABLE III
COMPARISON OF RESULTS OBTAINED THROUGH SIMULATION AND GEV
PARAMETERS FOR $\sigma_{SH} = 7.5$ dB.

Simulation Configuration		Dissimilarity Ξ	Percentage w.r.t max error
P_{Tx} [dBm]	R [m]		
43	1000	0.095	4.73
43	1250	0.073	3.65
43	1500	0.056	2.83
43	1750	0.058	2.92
43	2000	0.1	5
40	1500	0.065	3.27
46	1500	0.075	3.77

Fig. 5. MCS Probabilities for $\sigma_{SH} = 7.5$ dB and reuse 3x3x3.

maximum value of R is considered to be 2000 m beyond which outage probability increases rapidly [14]. PDFs and MCS probabilities are obtained through simulations with different configurations are compared with those obtained through GEV parameters.

The results of validation and applicability for various cell configurations are given in Fig. 5 and Tab. III. For MCS probabilities, maximum difference was found to be 0.06 (for MCS 64QAM-3/4) with simulation configuration of $R = 1000$ m, $P_{Tx} = 43$ dBm, which is 13% of the value of MCS 64QAM-3/4 probability. As far as PDF error is concerned, the percentage error w.r.t maximum possible error never exceeds 5% for all cell configurations.

This can be explained by the fact that the considered environment is dominated by interference, whereas thermal noise has a small effect. In this context, the network is homothetic and variations of the cell range and/or of the BS transmit power has a little impact on the SINR.

IV. TRAFFIC ANALYSIS

In this section, we provide an overview of the analytical model we use to perform the traffic analysis. Our model, especially developed for performance evaluation in WiMAX networks, focuses on the traffic belonging to the Best-Effort service class. As our main concern is to introduce the various parameters needed for the dimensioning procedure, we won't detail their expressions. However, note that they have already been fully detailed, explained and validated in [15]–[17].
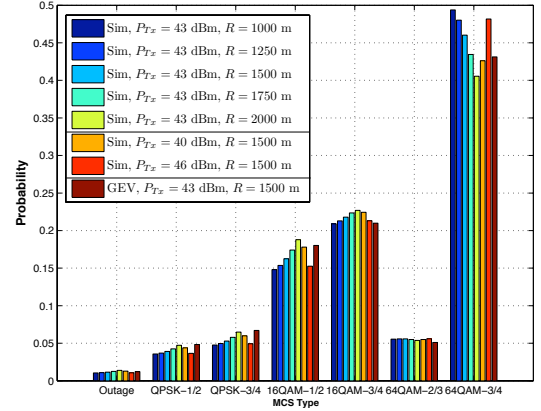
A. Modeling Assumptions

Our model stands on several assumptions related to the system, the channel, the traffic and the scheduling algorithm. Those assumptions have been presented, discussed and validated in [15]. Here we recall them and introduce various notations.

System Assumptions
The duration of a WiMAX TDD frame is $T_F = 5$ms.

1) We consider a single WiMAX cell and focus on the downlink part which is a critical portion of asymmetric data traffic.
2) We assume that amount of overhead in the TDD frame is fixed. As a consequence, the total number of slots available for data transmission in the downlink part is constant and equals N_S.
3) The number of simultaneous mobiles that can be multiplexed in one TDD frame is not limited. As a consequence, any connection demand will be accepted and no blocking can occur.

Channel Assumption
We denote the radio channel states as: MCS_k, $1 \leq k \leq K$, where K is the number of MCS. By extension, MCS_0 represents the outage state. The number of bits transmitted per slot by a mobile station (MS) using MCS_k is denoted by

m_k.

4) The coding scheme used by a given mobile can change very often because of the high variability of the radio link quality. We assume that each mobile sends a feedback channel estimation on a frame by frame basis, and thus, the base station can change its coding scheme every frame. We associate a probability p_k with each coding scheme MCS_k, and assume that, at each time-step T_F, any mobile has a probability p_k to use MCS_k.

Traffic Assumptions

5) We assume that there is a fixed number N of mobiles that are sharing the available bandwidth of the cell.

6) Each of the N mobiles is assumed to generate an infinite length ON/OFF elastic traffic. An ON period corresponds to the download of an element (e.g., a web page). The downloading duration depends on the system load and the radio link quality, so ON periods must be characterized by their size. An OFF period corresponds to the reading time of the last downloaded element, and is independent of the system load. As opposed to ON periods, OFF periods must then be characterized by their duration.

7) We assume that both ON sizes and OFF durations are exponentially distributed. We denote by $\bar{x}_{on}$ the average size of ON data volumes (in bits) and by $\bar{t}_{off}$ the average duration of OFF periods (in seconds).

Scheduling Assumption

Several scheduling schemes can be considered. In [15], we focused on three traditional schemes:

- The slot sharing fairness scheduling equally divides the slots of each frame between all active users that are not in outage.
- The instantaneous throughput fairness scheduling shares the resource in order to provide the same instantaneous throughput to all active users not in outage.
- The opportunistic scheduling gives all the resources to active users having the highest transmission bit rate, i.e., the better MCS.

Lastly, in [17], we proposed an alternative scheduling which forces an upper bound on the users' throughputs, the maximum sustained traffic rate (MSTR):

- The throttling scheduling tries to allocate at each frame the right number of slots to each active mobile in order to achieve its MSTR. If a mobile is in outage it does not receive any slot and its throughput is degraded. If at a given time the total number of available slots is not enough to satisfy the MSTR of all active users (not in outage), they all see their throughputs equally degraded.

B. Mono-Traffic Model

As a first step, we do not make any distinction between users and consider all mobiles as statistically identical. Thus, we consider that the N users are generating infinite-length ON/OFF Best-Effort traffics with the same traffic profile $(\bar{x}_{on}, \bar{t}_{off})$.

We model this system by a continuous-time Markov chain (CTMC) where each state n, represents the total number of concurrent active mobiles, regardless of the coding scheme they use. So, the resulting CTMC is made of $N + 1$ states as shown in Fig 6.
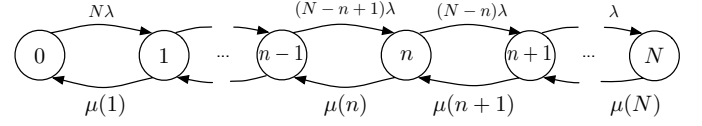
Fig. 6. General CTMC with state-dependent departure rates.

- A transition out of a generic state n to a state $n + 1$ occurs when a mobile in OFF period starts its transfer. This "arrival" transition corresponds to one mobile among the $(N - n)$ in OFF period, ending its reading, and is performed with a rate $(N - n)\lambda$, where λ is defined as the inverse of the average reading time: $\lambda = \frac{1}{\bar{t}_{off}}$.
- A transition out of a generic state n to a state $n - 1$ occurs when a mobile in ON period completes its transfer. This "departure" transition is performed with a generic rate $\mu(n)$ corresponding to the total departure rate of the frame when n mobiles are active.

Obviously, the main difficulty of the model resides in estimating the aggregate departure rates $\mu(n)$.

If we consider either the instantaneous throughput fairness, the slot sharing fairness or the opportunistic policy, they are expressed as follows:

$$\mu(n) = \frac{\bar{m}(n)\, N_S}{\bar{x}_{on}\, T_F}, \tag{2}$$

where $\bar{m}(n)$ is the average number of bits transmitted per slot when there are n concurrent active transfers. Note that these parameters are strongly dependent on the scheduling policy. As a consequence, we provide their expression depending on the considered policy.

With the slot sharing policy:

$$\bar{m}(n) = \sum_{\substack{(n_0, ..., n_K) = (0, ..., 0)\,| \\ n_0 + ... + n_K = n \\ n_0 \neq n}}^{(n, ..., n)} \frac{n!}{n - n_0} \sum_{k=1}^{K} m_k n_k \prod_{k=0}^{K} \frac{p_k^{n_k}}{n_k!}. \tag{3}$$

With the instantaneous throughput fairness policy:

$$\bar{m}(n) = \sum_{\substack{(n_0, ..., n_K) = (0, ..., 0)\,| \\ n_0 + ... + n_K = n \\ n_0 \neq n}}^{(n, ..., n)} \frac{(n - n_0)\, n! \prod_{k=0}^{K} \frac{p_k^{n_k}}{n_k!}}{\sum_{k=1}^{K} \frac{n_k}{m_k}}. \tag{4}$$

With the opportunistic policy:

$$\bar{m}(n) = \sum_{k=1}^{K} m_k\, \alpha_k(n). \tag{5}$$

where $\alpha_k(n)$ is the probability of having at least one active user (among n) using MCS_k and none using a better MCS:

$$\alpha_k(n) = \left(1 - \sum_{j=k+1}^{K} p_j\right)^n \left(1 - \left(1 - \frac{\frac{p_k}{k}}{\sum_{j=0} p_j}\right)^n\right). \quad (6)$$

If we consider the throttling policy, the departure rates $\mu(n)$ become:

$$\mu(n) = \frac{N_S}{max\,(n\bar{g}, N_S)} n \frac{MSTR}{\bar{x}_{on}}. \quad (7)$$

with $\bar{g}$, the average number of slots per frame needed by a mobile to obtain its MSTR:

$$\bar{g} = T_F\, MSTR \sum_{k=1}^{K} \frac{p_k}{(1 - p_0)m_k}. \quad (8)$$

Once the departure rates $\mu(n)$ have been determined, the steady-state probabilities $\pi(n)$ of having n concurrent transfers in the cell, can easily be derived from the birth-and-death structure of the Markov chain:

$$\pi(n) = \left(\prod_{i=1}^{n} \frac{(N - i + 1)\lambda}{\mu(i)}\right) \pi(0), \quad (9)$$

where $\pi(0)$ is obtained by normalization.

The performance parameters of the system can be obtained from the steady-state probabilities as follows. The average number of active users $\bar{Q}$ is expressed as:

$$\bar{Q} = \sum_{n=1}^{N} n\,\pi(n), \quad (10)$$

and $\bar{D}$, the mean number of departures (i.e., mobiles completing their transfer) per unit of time, is obtained as:

$$\bar{D} = \sum_{n=1}^{N} \mu(n)\,\pi(n). \quad (11)$$

From Little's law, we can thus derive the average duration $\bar{t}_{on}$ of an ON period (duration of an active transfer):

$$\bar{t}_{on} = \frac{\bar{Q}}{\bar{D}}. \quad (12)$$

and compute the average throughput $\bar{X}$ obtained by each mobile in active transfer as:

$$\bar{X} = \frac{\bar{x}_{on}}{\bar{t}_{on}}. \quad (13)$$

Finally, we can express the average utilization $\bar{U}$ of the TDD frame. This last parameter depends on the scheduling policy. Indeed, with the instantaneous throughput fairness, the slot sharing fairness or the opportunistic policy, the cell is considered fully utilized as long as there is at least one active mobile not in outage:

$$\bar{U} = \sum_{n=1}^{N} (1 - p_0^n)\pi(n). \quad (14)$$

However, if we consider the throttling policy, U is now expressed as the weighted sum of the ratios between the mean number of slots needed by the n mobiles to reach their MSTR and the mean number of slots they obtain:

$$\bar{U} = \sum_{n=1}^{N} \frac{n\bar{g}}{max\,(n\bar{g}, N_S)}\pi(n). \quad (15)$$

Lastly, let us remind that more detailed explanations on the models and the different relations presented here are available in [15] and [17].

C. Multi-Traffic Extension

Now, we relax the assumption that all users have the same traffic profile. To this aim, we associate to each mobile one of the R traffic profiles, $(\bar{x}_{on}^r, \bar{t}_{off}^r)$. The mobiles of a given profile r thus generate an infinite-length ON/OFF traffic, with an average ON size of $\bar{x}_{on}^r$ bits and an average reading time of $\bar{t}_{off}^r$ seconds. We assume that there is a fixed number N_r of mobiles belonging to each profile in the cell. As a consequence, there are $N = \sum_{r=1}^{R} N_r$ users in the cell with different traffic profiles.

To compute the performance parameters, we first transform this system into an equivalent one where all profiles of traffic have the same average ON size $\bar{x}_{on}$, and different average OFF durations $\bar{t'}_{off}^r$, such that:

$$\frac{\bar{x}_{on}}{\bar{t'}_{off}^r} = \frac{\bar{x}_{on}^r}{\bar{t}_{off}^r}. \quad (16)$$

With this transformation, the equivalent system can be described as a mutli-class closed queuing network with two stations as shown by Fig. 7:
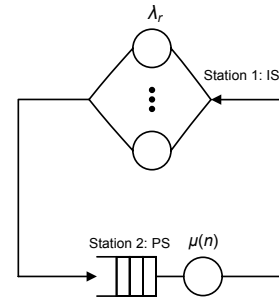
Fig. 7. Closed-queueuing network.

1) An IS station that models mobiles in OFF periods. This station has profile-dependent service rates $\lambda_r = \frac{1}{\bar{t'}_{off}^r}$;
2) A PS station that models active mobiles. This station has profile-independent service rates $\mu(n)$ that in turn depend on the total number active mobiles (whatever their profiles). They are given by the same relations than the departure rates of the mono-traffic model (see relations 2 and 7)

A direct extension of the BCMP theorem [18] for stations with state-dependent rates can now be applied to this closed

(a) Average number of active transfers (b) Average throughput per user (c) Average utilization of the cell

Fig. 8. Customary performance parameters in function of N and for each of the 4 scheduling policies.

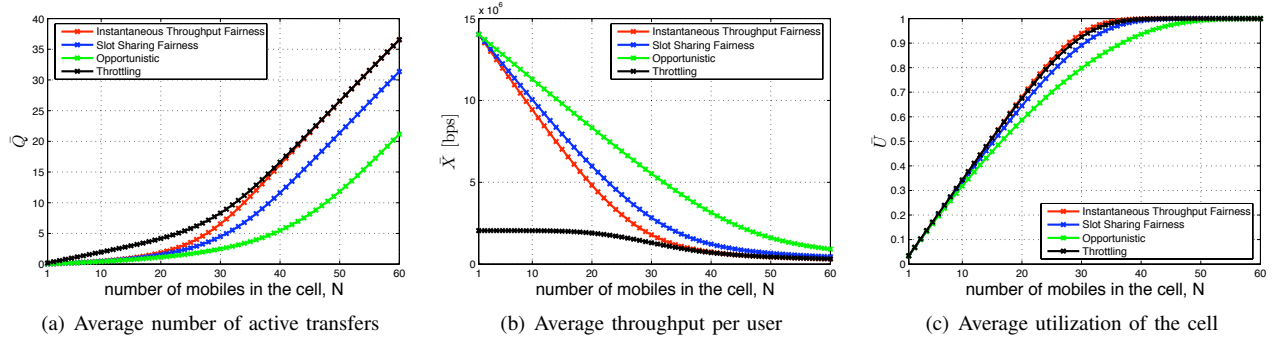queueing network. The detailed steady-state probabilities are expressed as follows:

$$\pi(\overrightarrow{n_1}, \overrightarrow{n_2}) = \frac{1}{G} f_1(\overrightarrow{n_1}) f_2(\overrightarrow{n_2}), \qquad (17)$$

where $\overrightarrow{n_i} = (n_{i1}, ..., n_{iR})$, n_{ir} is the number of profile-r mobiles present in station i,

$$f_1(\overrightarrow{n_1}) = \frac{1}{n_{11}!...n_{1R}!} \frac{1}{(\lambda_1)^{n_{11}}...(\lambda_R)^{n_{1R}}} \qquad (18)$$

$$f_2(\overrightarrow{n_2}) = \frac{(n_{21} + ... + n_{2R})!}{n_{21}!...n_{2R}!} \frac{1}{\prod_{k=1}^{n_2} \mu(k)}, \qquad (19)$$

and G is the normalization constant:

$$G = \sum_{\overrightarrow{n_1}+\overrightarrow{n_2}=\overrightarrow{N}} f_1(\overrightarrow{n_1}) f_2(\overrightarrow{n_2}). \qquad (20)$$

All the performance parameters of interest can be derived from the steady-state probabilities as follows. The average number of profile-r active mobiles, $\bar{Q}_r$, is given by:

$$\bar{Q}_r = \sum_{\overrightarrow{n_1}+\overrightarrow{n_2}=\overrightarrow{N}} n_{2r}\, \pi(\overrightarrow{n_1}, \overrightarrow{n_2}), \qquad (21)$$

and the average number of profile-r mobiles completing their download by unit of time, $\bar{D}_r$, can be expressed as:

$$\bar{D}_r = \sum_{\overrightarrow{n_1}+\overrightarrow{n_2}=\overrightarrow{N}} \mu(n_2)\, \pi(\overrightarrow{n_1}, \overrightarrow{n_2}), \qquad (22)$$

with $n_2 = \sum_{r=1}^{R} n_{2r}$.

The average download duration of profile-r mobiles, $\bar{t}_{on}^r$, is obtained from Little law:

$$\bar{t}_{on}^r = \frac{\bar{Q}_r}{\bar{D}_r}. \qquad (23)$$

And we can then calculate the average throughput obtained by customers of profile r during their transfer, denoted by $\bar{X}_r$, as:

$$\bar{X}_r = \frac{\bar{x}_{on}^r}{\bar{t}_{on}^r} \qquad (24)$$

Finally, the utilization $\bar{U}$ of the TDD frame is expressed differently whether we consider the instantaneous throughput fairness, the slot sharing fairness, the opportunistic policy:

$$\bar{U} = \sum_{\overrightarrow{n_1}+\overrightarrow{n_2}=\overrightarrow{N}} (1 - p_0^{n_2})\pi(\overrightarrow{n_1}, \overrightarrow{n_2}), \qquad (25)$$

or the throttling policy:

$$\bar{U} = \sum_{\overrightarrow{n_1}+\overrightarrow{n_2}=\overrightarrow{N}} \frac{\bar{g}(n_2)}{max\,(\bar{g}(n_2), N_S)} \pi(\overrightarrow{n_1}, \overrightarrow{n_2}). \qquad (26)$$

Again, fully detailed explanations on the multi-traffic model and the different relations are available in [16] and [17].

D. Numerical Results

Numerical Results are presented in this section, first for a mono-traffic scenario, then for a multi-traffic one. The channel parameters, identical in both cases, are summarized in Table IV. The probabilities p_k of using MCS_k correspond to the ones obtained in Section III-C and shown in Figure 5.

TABLE IV
CHANNEL PARAMETERS.

Index k	MCS	bits per slot m_k	probabilities p_k
0	Outage	0	0.020
1	QPSK 1/2	48	0.050
2	QPSK 3/4	72	0.070
3	16QAM 1/2	96	0.175
4	16QAM 3/4	144	0.210
5	64QAM 2/3	192	0.050
6	64QAM 3/4	216	0.425

1) Mono-traffic scenario: Table V gives the cell and traffic parameters of the mono-traffic scenario. Figure 8 illustrates the evolution of the $\bar{Q}$, $\bar{X}$ and $\bar{U}$ performance parameters when N increases for the 4 scheduling policies. Obviously, when N rises, so does the traffic load. As a consequence, the performances worsen as shown in Figure 8. Also, comparing the perfomances in function of the scheduling policy, we observe that the opportunistic policy alway gives the best results while the throttling policy does the exact opposite.

2) Multi-traffic scenario: The cell and traffic parameters describing the multi-traffic scenario are summarized in Table VI. We show in Figure 9 the evolution of the $\bar{Q}_r$, $\bar{t}_{on}^r$ and $\bar{U}$ performance parameters when N increases. Again, we can observe the performances getting worse as the load rises. Moreover, note that this phenomenon reaches all the users in the cell whatever their traffic profile.
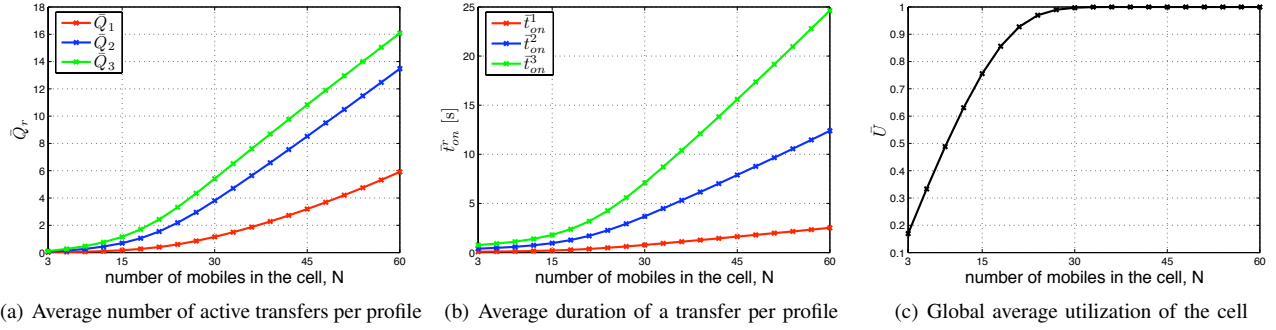
(a) Average number of active transfers per profile   (b) Average duration of a transfer per profile   (c) Global average utilization of the cell

Fig. 9. Customary performance parameters in function of N.

TABLE V
MONO-TRAFFIC SCENARIO.

Parameter	Value
Number of slots per trame, N_S	450 slots
Duration of a trame, T_F	5 ms
Number of MS in the cell, N	1 to 60 MS
Mean size of ON data volumes, $\bar{x}_{on}$	3 Mbits
Mean duration of OFF periods, $\bar{t}_{off}$	6 s
Scheduling policy	instantaneous throughput fairness, slot sharing fairness, opportunistic or throttling ($MSTR = 2$ Mbps)

TABLE VI
MULTI-TRAFFIC SCENARIO.

	Parameter	Value
	Number of slots per trame, N_S	450 slots
	Duration of a trame, T_F	5 ms
	Number of MS in the cell, N	3 to 60 MS
profile 1	Proportion of MS, α_1	1/3
	Mean size of ON data volumes, $\bar{x}_{on}^1$	1 Mbits
	Mean duration of OFF periods, $\bar{t}_{off}^1$	6 s
profile 2	Proportion of MS, α_2	1/3
	Mean size of ON data volumes, $\bar{x}_{on}^2$	5 Mbits
	Mean duration of OFF periods, $\bar{t}_{off}^2$	6 s
profile 3	Proportion of MS, α_3	1/3
	Mean size of ON data volumes, $\bar{x}_{on}^3$	10 Mbits
	Mean duration of OFF periods, $\bar{t}_{off}^3$	6 s
	Scheduling policy	slot sharing fairness

V. DIMENSIONING

In Section III, we explained how to determine the stationary MCS probabilities p_k. Then, we presented in Section IV the analytical model using those probabilities to perform the traffic analysis. Now, we show through an example how the model can be used to dimension any parameters of a WiMAX cell and present numerical results obtained with our method.

A. Procedure

Our analytical model provides closed-form expressions for all the performance parameters of interest. As such, we can obtain them for any configurations almost instantaneously through the resolution of the model. So, in order to dimension a given parameter of the WiMAX cell, we first establish a set of conditions we want to guarantee on the various performance parameters. Then, we just reiterate the resolution of the model while increasing this parameter until at least one of the dimensioning conditions is violated.

The following algorithm details this procedure when the parameter to dimension is the number of users in the cell, N, knowing the proportions α_r of mobiles generating traffic of profile r.

Require: all the cell parameters other than N.
    $n \leftarrow 0$.
    $(N_1, ..., N_R) \leftarrow (0, ..., 0)$.
    for all traffic profiles r **do**
        Transform $(\bar{x}_{on}^r, \bar{t}_{off}^r)$ into $(\bar{x}_{on}, \bar{t}'^r_{off})$ (relation 16).
    end for
    repeat
        $n \leftarrow n + 1$.
        if $(N_1, ..., N_R) \neq (\lfloor n\, \alpha_1 \rfloor, ..., \lfloor n\, \alpha_R \rfloor)$ **then**
            $(N_1^{max}, ..., N_R^{max}) \leftarrow (N_1, ..., N_R)$.
            $(N_1, ..., N_R) \leftarrow (\lfloor n\, \alpha_1 \rfloor, ..., \lfloor n\, \alpha_R \rfloor)$.
            Compute the departure rates $\mu(n)$ (2) or (7) depending on the scheduling policy.
            Compute the steady states probabilities $\pi(\overrightarrow{n_1}, \overrightarrow{n_2})$ (17).
            Compute the performance parameters (21) to (26).
        end if
    until at least one of the dimensioning conditions is violated.
    $N^{max} \leftarrow \sum_{r=1}^{R} N_r^{max}$
    return N^{max}, the maximum number of mobiles that can be allowed in the cell while respecting the dimensioning conditions.

Note that this very simple algorithm is only possible thanks to the instantaneous resolution of our analytical model. Indeed, this procedure would prove to be completely intractable with simulations due to the very long processing time they would require at each iteration of the loop.

B. Numerical Results

Let us consider again the multi-traffic scenario introduced in Section IV-D. Tables IV and VI summarize the channel and system parameters.

In this specific configuration, what is the maximum number of mobiles, N^{max}, we can have in the cell while still guaranteeing that the average utilization $\bar{U}$ does not exceed 80% ? To answer this question, we just have to use the previous

algorithm. Doing so, we compute the first points of Figure 9(c) going from $N = 3$ to $N = 18$. This last point violates the condition set on $\bar{U}$. Thus, we deduce that $N^{max} = 15$ is the maximum number of mobiles we are looking for.

For which maximum number of mobiles, N^{max}, in the cell can we guarantee that the average duration of a transfer of a user of profile 3, $\bar{t}_{on}^3$ does not exceed 15 s ? Again, using the previous algorithm we compute the first points of the curve corresponding to profile 3 shown in Figure 9(b) until we reach $N = 45$ MS which corresponds to $\bar{t}_{on}^3 = 15.6$ s. From this last result, we conclude that $N^{max} = 42$ MS, corresponding to $\bar{t}_{on}^3 = 13.8$ s, is the maximum number of mobiles we can have in the cell while respecting the dimensioning condition.

Lastly, let us emphasize that these are only two examples among the infinite set of dimensioning questions that can be answered almost instantaneously with our method.

VI. CONCLUSION

In this paper, we propose a complete dimensioning process for OFDMA networks. This process starts from a radio coverage analysis which aim is to furnish MCS probabilities to a traffic analysis module. The radio coverage analysis is usually obtained through extensive simulations. In this paper, we propose a semi-analytical method in order to reduce the computational time. This method is based on an approximation of the SINR spatial distribution by a GEV PDF. The traffic analysis is based on a markovian approach and provides dimensioning parameters at a click speed. Closed-form formulas take into account the channel variations, the elastic nature of data traffic, the scheduling policy and allows the definition of multiple user profiles. Radio coverage and traffic analysis proposed in this paper form a turnkey solution for the dimensioning of OFDMA networks.

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