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RESEARCH ARTICLE

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Christelle Garnier[1]*, Yves Delignon[1], Hassan El Ghazi[2] and François Septier[1]

[1] Institut TELECOM/TELECOM Lille 1 - LAGIS UMR CNRS 8219, Cite Scientifique, BP 20145, 59653 Villeneuve d'Ascq Cedex, France
[2] Institut National des Postes et Telecommunications, Avenue Allal Al Fassi, Madinat Al Irfane, Rabat, Morocco

ABSTRACT

In this paper, we address the problem of multiple access interference (MAI) in a downlink multicarrier code division multiple access system. Because in the realistic case of correlated faded subcarriers, MAI greatly depends on the codes assigned to users, one way of improving performance without increasing receiver complexity consists in an appropriate code selection. We propose a code allocation strategy with the following properties: possible use with any code sequence and equalizer, low complexity, and efficient management of load variations. The allocation problem is formulated as a minimization problem of a cost function related to MAI. First, we provide analytical expressions for the channel frequency correlation function after maximum ratio combining, equal gain combining, and minimum mean square error equalization and for the MAI power. Then, by approximating these expressions, we define a simple cost function and build an iterative algorithm on the basis of the minimum maximum criterion to select the spreading codes. Finally, a complete analysis of the allocation efficiency is provided versus key parameters, in particular, the degree of correlation between the faded subcarriers, the system load, and the equalization techniques. Copyright © 2012 John Wiley & Sons, Ltd.

1. INTRODUCTION

Multicarrier code division multiple access (MC-CDMA) [1,2] is a promising solution for the physical layer of future multiuser wireless networks, such as 4G cellular networks [3,4], broadband wireless access [5], and cognitive radio systems [6,7], especially for the downlink transmission. With the combination of orthogonal frequency division multiplexing (OFDM) and CDMA, it benefits from the advantages of both techniques, in particular, high data rate transmission capability in multipath environments, flexibility in spectrum use, and large multiple access capacity. Unlike CDMA, MC-CDMA performs the spreading operation in the frequency domain, mapping the spreading code chips on different subcarriers. Thus, it introduces diversity, but in a frequency selective fading channel, the orthogonality between user codes is destroyed, and

the resulting multiple access interference (MAI) severely limits the system performance.

To mitigate MAI in MC-CDMA systems, recent research has focused on several approaches including multiuser detection (MUD), band allocation, and spreading code selection. MUD techniques [2,8,9] are known to be effective, but the computational complexity strongly increases with the number of active users, so the implementation is not feasible in a mobile device. The principle of band allocation [10,11] is to appropriately divide the subcarriers into groups forming independent MC-CDMA systems. MAI is canceled at the receiver by applying MUD per group, which requires a lower computation capacity, but the number of supportable users is reduced.

Because in the realistic case of correlated faded subcarriers, the MAI of a downlink MC-CDMA system greatly depends on the spreading codes assigned to users,

another approach consists in a judicious code selection. The advantage is that the MC-CDMA architecture is not modified and a minimum amount of computation, which can use simple single-user detection (SUD), is required at the receiver. The issue is then to find the best spreading code families in a statistical sense [12–17] and the optimal set of codes to assign to the active users for a given family [18–25].

This paper deals with the problem of the optimal code set selection in a downlink MC-CDMA system. It is essentially an optimization problem, and several solving techniques have been proposed in literature. They differ in the cost function, which is related to the MAI term itself or to the MAI power and involves or does not involve the channel knowledge. They also use different minimization procedures: derivation of the exact solution (only for Walsh codes), exhaustive or iterative search.

When the MAI term is directly considered as the cost function, the knowledge of channel state information (CSI) is needed. In [22], the authors do without this knowledge by demonstrating that a synchronous MC-CDMA system with maximum ratio combining (MRC) detection is MAI free whatever the channel state is if it uses a specific set of Walsh codes. The drawback is that the maximum size of this set is the ratio between the number of subcarriers and the channel length, which limits the number of MAI free users as in [10]. According to [25], this number is also influenced by the distribution of time delay differences among the channel paths. In [23], CSI is estimated by the simple least squares method, and then an iterative procedure is used to select the code set that minimizes the total MAI. The technique provides good results, but additional signaling is required to return downlink CSI to the base station, and the code set must be updated and reassigned to all active users whenever the channel of one user varies.

In other techniques, the cost function is based on the MAI power whose expression involves the statistical properties of the channel as the frequency correlation function (FCF) after equalization. In a simplification purpose, several selection algorithms use indicators of the MAI power that only depend on spreading codes and their variability. In [18,19], an exhaustive search is performed to select the codes with a maximum number of sign changes, and in [20], an easier approach, specific to Walsh sequences, consists in directly choosing the codes with a maximum frequency separation among spectra. Both methods improve the system performance, but they do not provide the optimal code sets. In [21], the authors got better results by assuming the FCF of the equalized channel is perfectly known and by using an iterative process for minimizing the MAI power, but they provided no analytical expression and no estimation method.

In this paper, we propose a new method for selecting the optimal code set in a downlink MC-CDMA transmission through a correlated Rayleigh fading channel. On the basis of MAI power minimization, it takes into account the channel statistical properties accurately via the exact calculation of the FCF after equalization. For the

channel, we assume an exponential power delay profile (PDP), which is the most widely accepted PDP model in accordance with theory and experimental data [26]. For equalization, the common SUD techniques including MRC, equal gain combining (EGC), and minimum mean square error (MMSE) are considered. The cost function used in the minimization process is derived from an approximation of the exact expression of the FCF without performance loss. Because there is no analytical solution and the exhaustive search suffers from a prohibitive computational cost, the proposed method uses an iterative solving procedure based on the minimum maximum criterion. It is more efficient than the minimum average criterion and guarantees that no communication is disadvantaged by strong interference. The method also presents the following advantages: application to any code family and any equalization technique, low computational cost, and efficient management of load variations, which occur very frequently in a multiuser wireless network. At last, the performance gain achieved by code selection is improved if the MC-CDMA system controls both the spreading code set assigned by the base station and the SUD technique applied by the receiver.

The paper is organized as follows. Section 2 presents the MC-CDMA system model. Section 3 focuses on the expressions of the FCF of the equalized channel and of the MAI power. The cost function is then defined from an approximation of these expressions in Section 4, and an iterative algorithm is proposed to select the optimal spreading codes allocated to users. Section 5 presents simulation results including comparisons with existing allocation algorithms and with MUD techniques and a complete analysis of the allocation efficiency versus key parameters, in particular, the degree of correlation between the faded subcarriers, the system load, and the equalization techniques. Conclusions are finally drawn in Section 6.

Notation: N_c, N_s, and N_u respectively denote the spreading code length, the number of subcarriers, and the number of active users. The code assigned to user u is identified by the number z_u and is denoted c^{z_u}. T is the sampling time, T_s the duration of the OFDM symbol useful part and $\Delta f = 1/T_s$ the frequency spacing between adjacent subcarriers. $\binom{n}{k}$ represents the binomial coefficient, that is, the number of combinations of k from n, $(\cdot)^*$ denotes the complex conjugate operation and $\Re[\cdot]$ is the real part of a complex variable.

2. MULTICARRIER CODE DIVISION MULTIPLE ACCESS SYSTEM MODEL

We consider the downlink MC-CDMA system represented by Figure 1. It synchronously transmits through a channel assumed to be slowly time varying and frequency selective over the total bandwidth. In multicarrier systems, it is essential that each subcarrier experiences a flat fading, so Δf must be chosen shorter than the channel coherence
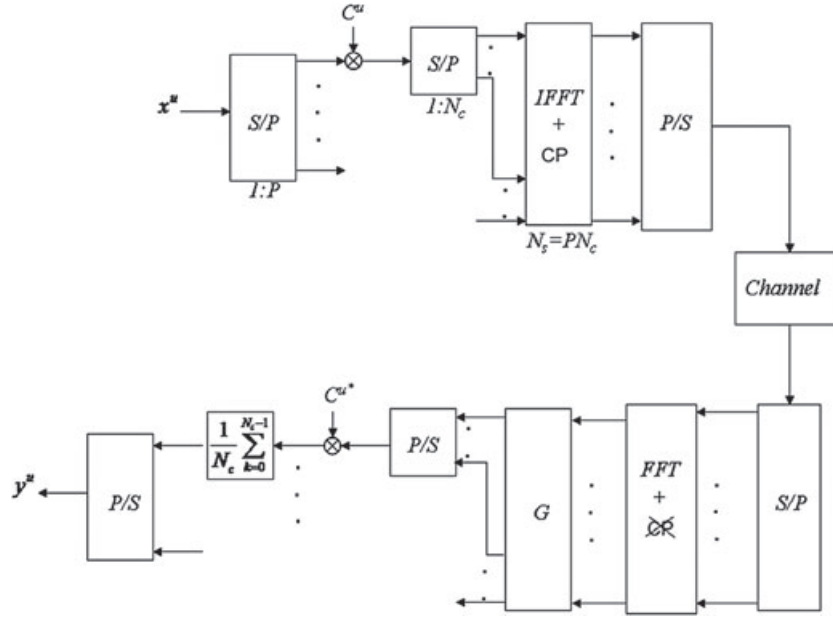
Figure 1. Block diagram of the MC-CDMA system for user *u*.

bandwidth. This requirement has two consequences. For high data rates, serial-to-parallel conversion must be performed before spreading, as proposed in [2]. Moreover the fading processes affecting the different subcarriers are necessarily correlated. In this section, we describe the transmitter, the receiver, and the channel models. In particular, we are interested in the channel FCF to quantify the correlation degree between the faded subcarriers of the MC-CDMA system.

2.1. Transmitter model

After serial-to-parallel conversion of P symbols, the pth symbol of user u x_p^u is spread with the orthogonal user code $c^{z_u} = \{c_k^{z_u}, k = 0, \dots, N_c - 1\}$, where $|c_k^{z_u}|^2 = 1$. The components of the spread sequence $x_p^u c_k^{z_u}$ are then transmitted in parallel on N_c subcarriers selected among a total amount of $N_s = P N_c$ orthogonal subcarriers. To achieve frequency diversity, the gap between the N_c subcarriers conveying the same symbol x_p^u is maximized; it is then equal to $P \Delta f$ as in [2]. Consequently, the chips $x_p^u c_k^{z_u}$ are sent on subcarriers $p + kP$ for $k = 0, \dots, N_c - 1$. After the inverse fast Fourier transform (FFT), the MC-CDMA symbol is extended with a cyclic prefix longer than the channel maximum excess delay. Considering all assumptions, the system is prevented from intersymbol and intersubcarrier interference. Afterwards, to simplify notations and without loss of generality, the index p is omitted. The subcarriers used to transmit the symbol denoted x^u are just identified by index k varying from 0 to $N_c - 1$.

2.2. Correlated channel model

We assume a wide-sense stationary uncorrelated scattering Rayleigh fading channel modeled as a tapped delay line filter. The impulse response is constant over the MC-CDMA symbol duration T_s and is written as

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - lT) \tag{1}$$

where L is the number of taps, h_l is the complex gain, T is the sampling time and lT is the delay of the lth tap.

We consider an exponential PDP characterized by a root mean square (RMS) delay spread σ_τ. It is the most commonly accepted PDP model for indoor channels in accordance with theory and experimental data [26]. When the maximum excess delay $(L - 1)T$ is large enough compared with σ_τ, the FCF of the channel model, obtained by Fourier transform of the PDP, is accurately approximated by [27]

$$R_H(f) = \frac{1 - e^{-\frac{T}{\sigma_\tau}}}{1 - e^{-(1 + j 2\pi f \sigma_\tau)\frac{T}{\sigma_\tau}}} \tag{2}$$

It is interesting to note that when T is also small enough compared with σ_τ, we get the FCF given by Jakes [26] corresponding to the theoretical exponential PDP (i.e., with continuous time and illimited duration). In the MC-CDMA system, the sampling time is $T = 1/(N_s \Delta f)$, and the frequency spacing between two subcarriers k and $k + n$ transmitting the same symbol is equal to

$nP\Delta f = n(N_s/N_c)\Delta f$. If we denote H_k the channel fading on subcarrier k, the FCF $R_H(n)$ between H_k and H_{k+n} is obtained by substituting these expressions in (2):

$$R_H(n) = E[H_k H_{k+n}^*] = \frac{1 - e^{-\frac{1}{N_s\Delta f \sigma_\tau}}}{1 - e^{-(\frac{1}{N_s\Delta f \sigma_\tau} + j2\pi\frac{n}{N_c})}}$$
(3)

The correlation degree between the faded subcarriers of the MC-CDMA system is characterized by the product of the system bandwidth $N_s\Delta f$ with the channel RMS delay spread σ_τ. The lower $N_s\Delta f \sigma_\tau$ is, the more the channel coefficients are correlated. If we consider realistic transmission conditions similar to IEEE 802.11 standard with a bandwidth of 20 MHz and RMS delay spreads between 15 and 150 ns depending on the environment [28], the values of $N_s\Delta f \sigma_\tau$ range from 0.3 to 3.

2.3. Receiver model

After cyclic prefix removal, OFDM demodulation by FFT, and SUD application, the decision variable y^u for user u can be expressed as the summation of three terms that respectively correspond to the useful signal, the MAI, and the noise:

$$y^u = x^u \frac{1}{N_c} \sum_{k=0}^{N_c-1} H_k g_k + x_{MAI}^{z_u} + \frac{1}{N_c} \sum_{k=0}^{N_c-1} c_k^{z_u*} g_k \eta_k$$
(4)

where g_k is the equalization coefficient on the kth subcarrier and η_k is the additive white Gaussian noise with power $N_u\sigma^2$ at the FFT output.

The MAI term results from the mutual interference $x_{MAI}^{z_u,z_v}$ between the detected user u and each interfering user v due to code orthogonality alteration by the channel and the equalizer:

$$x_{MAI}^{z_u} = \sum_{\substack{v=0 \\ v\neq u}}^{N_u-1} x_{MAI}^{z_u,z_v} = \sum_{\substack{v=0 \\ v\neq u}}^{N_u-1} x^v \frac{1}{N_c} \sum_{k=0}^{N_c-1} c_k^{z_u*} c_k^{z_v} H_k g_k$$
(5)

Several SUD techniques corresponding to different diversity combining strategies have been proposed in literature [2]. Table I summarizes the expressions of the equalization coefficient for MRC, zero forcing (ZF), EGC, and MMSE techniques. By channel inversion, ZF restores orthogonality and totally eliminates MAI, but the performance is degraded by noise amplification on subcarriers affected by deep fading. In the following, we are interested in MAI minimization to improve the

Table I. Expressions of the equalization coefficient.

Equalization method	MRC	ZF	EGC	MMSE
Expression of g_k	H_k^*	$\frac{1}{H_k}$	$\frac{H_k^*}{|H_k|}$	$\frac{H_k^*}{|H_k|^2+\frac{1}{\gamma}}$

$\gamma = \frac{1}{\sigma^2}$ is the signal-to-noise ratio at the output of the FFT.

MC-CDMA system performance, so only MRC, EGC, and MMSE equalizers are investigated.

3. FREQUENCY CORRELATION FUNCTION OF THE EQUALIZED CHANNEL, MULTIPLE ACCESS INTERFERENCE POWER, AND PROBLEM FORMULATION

From (5), the power of the MAI between two active users u and v is written as

$$P_{MAI}^{z_u,z_v} = \frac{1}{N_c^2} \sum_{k=0}^{N_c-1} \sum_{k'=0}^{N_c-1} R_{Hg}(k'-k) c_k^{z_u*} c_k^{z_v} c_{k'}^{z_u} c_{k'}^{z_v*}$$
(6)

with $R_{Hg}(n) = E[H_k g_k H_{k+n}^* g_{k+n}^*]$ the FCF of the equalized channel. This section first provides and analyzes the channel FCF after MRC, EGC, and MMSE equalization in the case of an exponential PDP. Then, the analytical expression of the MAI power is derived. Finally, the code selection problem is expressed in a very general way with the aim of designing an algorithm independently of the code family and of the equalizer.

3.1. Frequency correlation function of the equalized channel

Whatever the equalizer, according to the expressions of g_k in Table I, the coefficient $H_k g_k$ affecting the kth subcarrier can always be written as a function of $|H_k|$. Consequently, $R_{Hg}(n)$ can be derived from the expression (A.1) of the joint density of Rayleigh channel envelopes given by Jakes [26]. The calculation, detailed in Appendix A, leads respectively to expressions (A.2), (A.3), and (A.6) for MRC, EGC, and MMSE detection.

According to these expressions, $R_{Hg}(n)$ is related to the absolute square of the channel FCF before equalization (3). As $R_H(n)$ is Hermitian and periodic with period N_c, $R_{Hg}(n)$ is even and also periodic:

$$R_{Hg}(n) = R_{Hg}(-n) = R_{Hg}(N_c+n) = R_{Hg}(N_c-n)$$
(7)

Figure 2 represents the normalized FCF $R_{Hg}(n)/R_{Hg}(0)$ for the different equalizers and for the following values of $N_s\Delta f \sigma_\tau$: 0.16, 0.48, and 1.6. For all equalizers, we can consider that $N_s\Delta f \sigma_\tau = 0.16$ corresponds to highly correlated subcarriers, because the FCF is almost constant. For other values, when the subcarrier spacing n increases, the FCF decreases from its maximum value at $n = 0$ to be closer to a horizontal asymptote $y = R_{Hg}(N_c/2)$. The decay is all the more sharp because the product $N_s\Delta f \sigma_\tau$ is high. In MMSE case, it also depends on the signal-to-noise ratio $\gamma = 1/\sigma^2 = (2N_u/N_c)(E_b/N_0)$. The figure also shows that the range of values taken by the FCF depends on the equalization technique; it is larger with MRC and very tighter with MMSE.
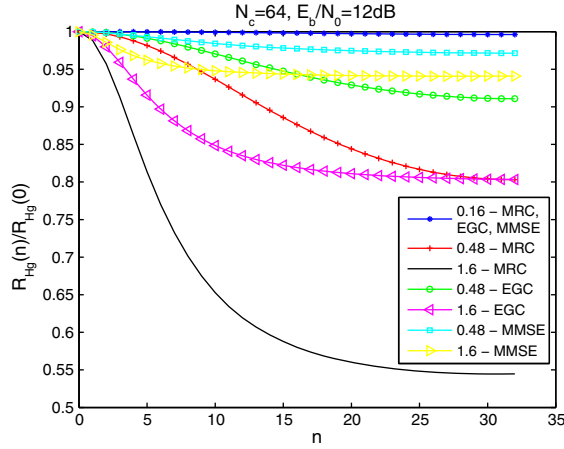
Figure 2. FCF of the equalized channel for MRC, EGC, and MMSE equalization and for several values of $N_s \Delta f \sigma_\tau$ ranging from 0.16 to 1.6.

3.2. Multiple access interference power

With the use of (7), the expression (6) of the MAI power between two users can be rewritten for an even value of N_c as

$$
P_{\text{MAI}}^{z_u, z_v} = R_{Hg}(0) + \frac{1}{N_c} R_{Hg}(N_c/2) \Re \left[R_\omega^{z_u, z_v}(N_c/2) \right]
$$
$$
+ \frac{2}{N_c} \sum_{n=1}^{N_c/2-1} R_{Hg}(n) \Re \left[R_\omega^{z_u, z_v}(n) \right] \quad (8)
$$

where $R_\omega^{z_u, z_v}(n)$ is the periodic correlation function (PCF) of the chip-by-chip product $\omega_k^{z_u, z_v} = c_k^{z_u*} c_k^{z_v}$ of the spreading codes of users u and v defined by

$$
R_\omega^{z_u, z_v}(n) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \omega_k^{z_u, z_v} \omega_{k+n}^{z_u, z_v*} \quad (9)
$$

Finally, the total MAI power depends on the following parameters: the number of users N_u, the code length N_c, the product $N_s \Delta f \sigma_\tau$, the ratio E_b/N_0 (for MMSE detection), the SUD technique, and the code product PCF governed by the spreading codes allocated to the active users. At every moment, the first parameters are fixed, whereas a not fully loaded MC-CDMA system has a degree of freedom to select N_u user codes among all available codes.

3.3. Code selection problem formulation

Code selection is a minimization problem that can be expressed as follows. Consider a spreading code family with N codes of length N_c identified by a number ranging from 0 to $N-1$ (in many cases, $N = N_c$). Let $S = \{0, 1, \ldots, N-1\}$ be the set of all codes and

$S_{N_u} = \{z_0, z_1, \ldots, z_{N_u-1}\} \subseteq S$ be a subset of N_u codes that can be allocated to the N_u active users of the MC-CDMA system. We look for the optimal code subsets $S_{N_u}^{(\text{opt})}$ that minimize a cost function $f(S_{N_u})$:

$$
S_{N_u}^{(\text{opt})} = \underset{S_{N_u}}{\arg \min} f(S_{N_u}) \quad (10)
$$

where $f(S_{N_u})$ is related to the MAI power $P_{\text{MAI}}^{z_u, z_v}$ between the different pairs of codes from S_{N_u}. Because the solution to the minimization problem is a priori not unique, we denote $\Omega_{N_u}^{(\text{opt})}$ the set including all code subsets $S_{N_u}^{(\text{opt})}$ leading to the minimum value.

According to expression (8), there is no analytical solution to (10). Furthermore, the exhaustive search among the $\binom{N}{N_u}$ combinations of codes requires a prohibitive amount of computations. Therefore, a simpler and faster solving technique is needed.

4. SPREADING CODE ALLOCATION STRATEGY

In order to achieve an acceptable computational cost, we propose a cost function that can be evaluated with few calculations and an iterative process to solve the minimization problem.

4.1. Proposed cost function

The proposed cost function is derived from an approximation of the equalized channel FCF and of the MAI power. According to expressions (A.2), (A.3), and (A.6), the FCF $R_{Hg}(n)$ of the equalized channel decreases from its maximum value at $n = 0$ to be closer to a horizontal asymptote $y = R_{Hg}(N_c/2)$ when n increases. To simplify expression (8), we suppose that from a gap d between the subcarriers, the equalized channel coefficients $H_k g_k$ have a minimum correlation: $R_{Hg}(n) \approx R_{Hg}(N_c/2) \ \forall n > d$.

Then, the power of the MAI between two users u and v is approximated by

$$
P_{\text{MAI}}^{z_u, z_v}(d) = R_{Hg}(0) + \frac{1}{N_c} R_{Hg}(N_c/2) \Re \left[R_\omega^{z_u, z_v}(N_c/2) \right]
$$
$$
+ \frac{2}{N_c} \sum_{n=1}^{d} R_{Hg}(n) \Re \left[R_\omega^{z_u, z_v}(n) \right]
$$
$$
+ \frac{2}{N_c} R_{Hg}(N_c/2) \sum_{n=d+1}^{N_c/2-1} \Re \left[R_\omega^{z_u, z_v}(n) \right]
$$
$$
\quad (11)
$$

$P_{\text{MAI}}^{z_u, z_v}(d)$ is the dth-order approximation of the MAI power between two users. As shown in Appendix B, the code product PCF verifies the following property for many

code sequences such as Walsh, orthogonal Gold, Golay, or carrier interferometry (CI):

$$\sum_{n=1}^{N_c-1} R_\omega^{z_u,z_v}(n) = -1 \qquad (12)$$

With the use of this property as well as the Hermitian nature and the periodicity with period N_c of the PCF, the dth-order approximation of the MAI power (11) becomes

$$P_{\text{MAI}}^{z_u,z_v}(d) = R_{Hg}(0) - R_{Hg}(N_c/2) + \frac{2}{N_c} \sum_{n=1}^{d}$$
$$\times \Re \left[R_\omega^{z_u,z_v}(n) \right] \left(R_{Hg}(n) - R_{Hg}(N_c/2) \right) \qquad (13)$$

Whatever the SUD technique and the approximation order, the deviation from the exact value of the MAI power is expected to decrease when $N_s \Delta f \sigma_\tau$ increases. Indeed, the FCF of the equalized channel $R_{Hg}(n)$ decreases more rapidly towards the asymptote, and the proposed approximation is more accurate. We can precise that the deviation is similar with MRC and EGC techniques. With MMSE-based detection, the deviation is smaller because of the lower range of values taken by the FCF between $n = 0$ and $n = N_c/2$; therefore, the proposed approximation is more accurate whatever the value of $N_s \Delta f \sigma_\tau$.

Finally, because in many applications, the minimum maximum criterion is known to be more efficient than the minimum average criterion [29,30], the cost function $f(S_{N_u})$ is defined as the maximum value of the dth-order approximation of the MAI power over all couples (z_u, z_v) from $S_{N_u} \times S_{N_u}$ rather than as the total MAI power. Among all possible code combination sets S_{N_u}, we look for the subsets that satisfy

$$S_{N_u}^{(d)} = \arg\min_{S_{N_u}} \max_{\substack{(z_u,z_v) \in S_{N_u} \times S_{N_u} \\ z_v < z_u}} P_{\text{MAI}}^{z_u,z_v}(d) \qquad (14)$$

With the maximum power minimized, no communication will be affected by strong interference; hence, the overall MAI will be kept at a low level. As previously, the solution to (14) is not necessarily unique, and we denote $\Omega_{N_u}^{(d)}$ the set including all code subsets $S_{N_u}^{(d)}$ leading to the minimum value.

4.2. Iterative algorithm

The principle of iterative search is to start from an initial state, here, a spreading code z_0 selected from the set S of the N available codes. Then, the following process is repeated: at each iteration k, we compare, for each of the $N - k$ remaining codes, the values of the approximate power of the MAI with the k codes already chosen in previous iterations. We select the remaining code z_k, which minimizes the maximum power over the first k codes.

After $N_u - 1$ iterations, we get a subset of N_u codes that represents a suboptimal solution. To improve the effectiveness of the method, the process is applied several times with different initial conditions. In total, N subsets of N_u codes $\left(S_{N_u}^{(d)} \right)_{n=1,\dots,N} = (z_0 = n, \dots, z_{N_u-1})$ are built from the N available codes. The final solution is the best of the suboptimal solutions, that is, the solution that meets the minimum maximum criterion. The two steps of the code allocation algorithm are described in detail in Table II.

It is interesting to note that the iterative nature of the algorithm makes easy and fast the reselection of codes when communications are starting or ending in the MC-CDMA system. Assume that the N subsets $\left(S_{N_u}^{(d)} \right)_{n=1,\dots,N} = \{z_0 = n, \dots, z_{N_u-1}\}$ built during the previous allocation of N_u codes are stored. Then, the selection of a new subset of codes only requires updating these N subsets and selecting the best one in accordance with the minimum maximum criterion. For this, one iteration must be performed for every initial state z_0 in the first step of the allocation algorithm. The objective is to select the missing codes z_{N_u} when the load changes from N_u to $N_u + 1$ active users and to suppress the last allocated codes z_{N_u-1} for a transition from N_u to $N_u - 1$ users. In both cases, the second stage of the algorithm remains unchanged.

4.3. Computational complexity

For an MC-CDMA system, given an environment, that is, a channel delay spread, the values of the MAI power can be precalculated from (13) for the $N_u(N_u - 1)/2$ distinct couples inside the code family. Then, to select an optimal set of N_u codes with the iterative algorithm, the cost function no longer needs to be evaluated for the $\binom{N}{N_u}$ combinations of codes but only for the N code subsets, which are built during the first step of the algorithm. At each evaluation, the maximum over all the power values is calculated. Then, the second stage of the algorithm presents a complexity equal to $\mathcal{O}\left(N N_u^2 \right)$. It must be added with the number of operations performed during the first step that builds the N subsets. Because for each of the N initial states, each iteration k consists in calculating a maximum over k values and a minimum over $N-k$ values, the complexity of this step is

$$\mathcal{O}\left(N \sum_{k=1}^{N_u-1} k(N-k) \right) = \mathcal{O}\left(N^2 N_u^2 \right) \qquad (15)$$

Finally, the total complexity of the iterative search algorithm is equal to $\mathcal{O}\left(N^2 N_u^2 \right)$. The prohibitive complexity of the exhaustive search is then reduced to an acceptable quadratic complexity versus N and N_u.

Table II. Code allocation algorithm.

First step: Construction of the N subsets of N_u codes

For $n = 0$ to $N - 1$

> The first code is initialized:
>
> $z_0 = n$
>
> **For** $k = 1$ to $N_u - 1$
>
> > The kth code is chosen to minimize the maximum value of the power
> > of the MAI between the selected and the remaining codes:
> >
> > $$z_k = \underset{\substack{z_u \in S \\ z_u \neq z_0, \cdots, z_{k-1}}}{arg\,min} \; \underset{z_v = z_0, \cdots, z_{k-1}}{max} P_{MAI}^{z_u, z_v}(d)$$
>
> **End for**
>
> The subset built from the initial state $z_0 = n$ is stored:
>
> $$\left(S_{N_u}^{(d)}\right)_n = \{z_0, \cdots, z_{N_u - 1}\}$$

End For

Second step: Selection of the optimal code subset

The optimal code subset among the N available subsets is the one which

minimizes the cost function:

$$S_{N_u}^{(d)} = \underset{\left(S_{N_u}^{(d)}\right)_n}{arg\,min} \; \underset{\substack{(z_u, z_v) \in \left(S_{N_u}^{(d)}\right)_n^2 \\ z_v < z_u}}{max} P_{MAI}^{z_u, z_v}(d)$$

Now, let us evaluate the number of operations required to obtain the $N_u(N_u - 1)/2$ values of the MAI power. According to (8) and (13), the exact and approximate expressions of the MAI power involve the PCF of the chip-by-chip product $R_\omega^{z_u, z_v}(n)$ and the FCF of the equalized channel $R_{Hg}(n)$. According to (9), the calculation of each value $R_\omega^{z_u, z_v}(n)$ requires $2N_c - 1$ operations. Referring to Appendix A, whatever the equalizer, each value $R_{Hg}(n)$ is related to the absolute square of the channel FCF before equalization $R_H(n)$, whose calculation requires four operations. Then, if we consider, for example, MRC equalizer (A.2), two operations are required to obtain $R_{Hg}(n)$, so the total number of operations is equal to six.

Finally, without approximation (8), the calculation of each power value requires N_c operations involving $N_c/2$ values of the code PCF $R_\omega^{z_u, z_v}(n)$ and $((N_c/2) + 1)$ values of the channel FCF $R_{Hg}(n)$. So the total number of operations is equal to $N_c + (N_c/2)(2N_c - 1) + 6((N_c/2) + 1)$, which corresponds to a quadratic complexity $\mathcal{O}(N_c^2)$. With the dth-order approximation (13), the calculation only requires $3d + 1$ operations involving d values for the PCF and $(d + 2)$ values for the FCF. So the total number of operations is reduced to $3d + 1 + d(2N_c - 1) + 6(d + 2)$, which corresponds to a linear complexity versus N_c: $\mathcal{O}(dN_c)$. The value of d can be chosen according to a trade-off between accuracy and complexity.

5. SIMULATION RESULTS

This section focuses on the performance gain obtained through our code allocation strategy. First, an MC-CDMA system with MRC detection is considered. We compare the performance results when using the code sets given by our algorithm and by other algorithms proposed in literature. Then, the bit error rate (BER) is evaluated for the different SUD schemes (MRC, EGC, and MMSE) versus key parameters including the load and the degree of correlation between the subcarriers. The section ends with a comparison with a simple MUD technique. For simulation, Walsh codes are considered because they provide good BER performance in a statistical sense among the most common code sequences [16,31]. The subcarriers are quadrature phase shift keying modulated, the code length N_c and the number of subcarriers N_s are equal to 64 as in IEEE 802.11a/g standard, and $E_b/N_0 = 12$ dB.

5.1. Comparison with other algorithms

Now, we compare the performance of an MC-CDMA system, which uses the code sets given by our algorithm and by other algorithms proposed in literature. Only the most effective techniques from previous publications

are investigated. They include the methods proposed by Tsai *et al.* [22], Mourad *et al.* [21], Jao *et al.* [23], and our algorithm. For both methods in [21] and [23], we consider ideal and realistic conditions, that is, perfect and imperfect channel estimation. For our algorithm, two approximation orders, which are accurate enough to select the optimal code sets and to obtain the minimum BER, are used for the MAI power: the 1st order and the 11th order. For reference, we also present the minimum BER obtained with the optimal set of codes (given by an exhaustive search) and the average BER obtained with a random selection of codes.

Figure 3 represents the BER versus the system load for $N_s \Delta f \sigma_\tau = 0.48$. It clearly shows that code allocation algorithms actually improve the performance of an MC-CDMA system.

When the number of users exceeds 16, the best performance equal to the minimum BER is achieved with all the techniques, even with the algorithms in [21] and [23] using imperfect channel estimation by the least squares method and with the proposed algorithm using the first-order approximation of the MAI power. For $N_u \leq 16$, the best allocation is still obtained with the algorithm in [22]. More accuracy is required for the other algorithms: perfect channel estimation for the algorithms in [21] and [23] and the 11th approximation order for our algorithm.

It is interesting to note that the performance evolution versus the number of users occurs by step. The same behavior can be observed in the algorithms in [22] and [23]. For $N_u \leq 16$, the BER obtained with the best code allocation remains constant and equals to 2×10^{-8}. This error floor corresponds to the optimal performance obtained for a single-user transmission, because the MC-CDMA system is MAI free and only affected by additive white Gaussian noise. It is an expected result because our algorithm provides exactly the same code subsets as

MRC, N_c=64, $N_s \Delta f \sigma_\tau$=0.48, E_b/N_0=12dB
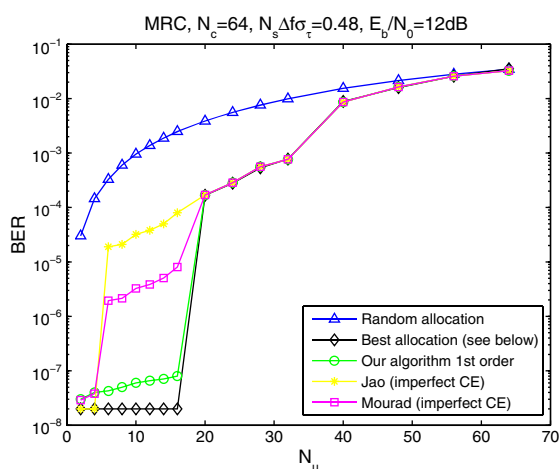
Figure 3. BER obtained with various allocations versus N_u. The "Best allocation" is obtained with the following algorithms: Tsai *et al.*, Mourad *et al.*, and Jao *et al.* with perfect channel estimation (CE) and our algorithm with the 11th-order approximation.

the method proposed by Tsai *et al.* whose aim is to make the MAI equal to zero in an MC-CDMA system using MRC equalization and Walsh codes. The authors demonstrated in [22] that the equation can be solved by using specific subsets of N_c/G codes, where G is a power of two with $G \geq L$. So these code subsets have the property of generating no MAI for a load $N_u \leq N_c/G$. Here, for $N_s \Delta f \sigma_\tau = 0.48$, the number of channel coefficients L is set to 4, so $G = 4$ and $N_c/G = 16$. Once the limit of $N_c/G = 16$ users is exceeded, the MAI-free property cannot be maintained, the new added codes generate interference with all the previous codes, and the performance strongly degrades. The BER is equal to 2×10^{-4} for $N_u = 20$ and then it slightly increases until $N_u = 32$. When the number of users exceeds this value, a strong degradation occurs again because additional codes with worse intercorrelation properties are used. Then, the BER weakly increases until fully loaded ($N_u = 64$).

For $N_u \leq 16$, the algorithms in [21] and [23] with imperfect channel estimation by the least squares method lead to a worse performance. They do not select the optimal code sets because of errors in channel estimates, and the BER quickly degrades. With our algorithm using the first-order approximation of the MAI power, the BER remains above the minimum BER, but the performance is better. In fact, when we analyze the code sets provided by our algorithm with the different approximation orders [30], we can observe that the following inclusion is always verified: $S_{N_u}^{(opt)} \subset \Omega_{N_u}^{(d)}$. So when the approximation order increases, the search for code sets is refined. Thus, the BER obtained with the first approximation order corresponds to the BER averaged on several code sets: all the optimal sets leading to the minimum BER and some other sets with a worse performance.

Finally, the proposed iterative algorithm always leads to the minimum BER when using the suitable approximation order. Compared with that of the algorithm in [22], the advantage of our algorithm is that it can apply to any code family and to any SUD scheme. Unlike the algorithm in [23], our algorithm uses channel statistical characteristics instead of CSI to select the best code subsets, so it requires no additional signaling (to return CSI to the base station) and no code reallocation to all the active users whenever the channel state of one user varies. Finally, compared with that of the algorithm in [21], the computational complexity is significantly reduced because our algorithm uses the analytical expressions of the FCF of the equalized channel, so the channel estimation stage is eliminated. It also uses the approximated MAI power instead of the exact power in the minimization process without performance loss.

5.2. Performance for the different equalizers

To quantify the performance gain for the different equalizers (EGC, MRC, and MMSE), we compare the

performance of an MC-CDMA system, which uses our code allocation algorithm and a random code allocation.

Figure 4 represents the BER versus the load for $N_s \Delta f \sigma_\tau = 0.48$. In our algorithm, two approximation orders are considered for the interference power: the 1st order and the 11th order which still leads to the minimum BER. The figure shows that the first-order approximation is accurate enough to provide the minimum BER in MMSE case whatever the load and in MRC and EGC cases for a load above 16. Nevertheless, for a lower load, it provides a BER smaller than 8×10^{-8} with MRC and than 4×10^{-7} with EGC, so it leads to a significant improvement of the performance.

Whatever the equalizer, the performance gain obtained with our code allocation strategy is very important for a loading rate smaller than 1/4, less important but still significant for a rate between 1/4 and 1/2, and strongly decreases when the rate exceeds 1/2 because two effects combine: the MAI level increases and the degree of freedom for choosing the N_u codes among the N available codes decreases.

More precisely in a low-loaded system, the MRC technique, which is the most sensitive to MAI, offers the most significant gain. For $N_u \leq N_c/G = 16$, MAI is completely avoided, and the performance remains constant, equal to 2×10^{-8} when using the code sets allocated by the algorithm with the 11th-order approximation. For comparison, for $N_u = 16$, the BER obtained with randomly selected codes is equal to 2.4×10^{-3}. However, once the limit of $N_c/G = 16$ users is exceeded, the performance strongly degrades. Even with code allocation, the MRC technique remains very sensitive to MAI. In EGC case, the performance gain is lower. We can observe the same per-step performance evolution, as described previously for MRC detection. But for $N_u \leq N_c/G = 16$, the MAI is not equal to zero, so the BER is above the BER obtained with MRC. The gaps between steps are also less important. The performance gain is much lower with the MMSE technique. For $N_u = 16$, the BER is equal to 3.8×10^{-6}
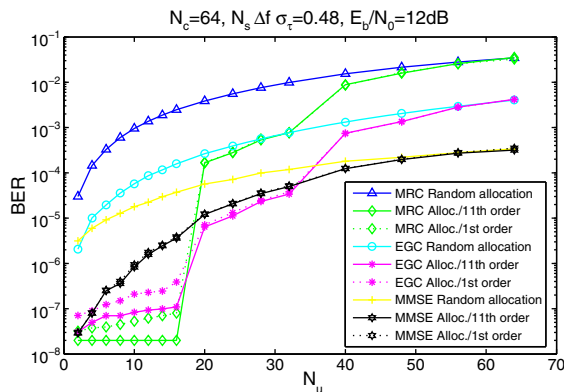
with the proposed allocation and to 3.7×10^{-5} with random allocation. This technique already partially restores the orthogonality of codes, so the set of codes allocated to users has less impact on performance.

Now in absolute, we can notice that the best performance is obtained when the allocation algorithm is coupled with MRC for a small load less than N_c/G, with EGC for a load between N_c/G and $N_c/2$, then with MMSE for a higher load.

Figure 5 shows the BER of a quarter-loaded MC-CDMA system versus $N_s \Delta f \sigma_\tau$, which characterizes the degree of correlation between subcarriers. In accordance with Figure 4, our code allocation algorithm uses the first-order approximation with MMSE scheme and the 11th-order approximation with MRC and EGC schemes to get the minimum BER.

We can notice that the performance gain is more important for low values of $N_s \Delta f \sigma_\tau$, that is, when the faded subcarriers are correlated. For example, for $N_s \Delta f \sigma_\tau = 0.48$, the average BER is respectively equal to 2.4×10^{-3}, 1.5×10^{-4}, and 3.8×10^{-5} with random allocation in MRC, EGC, and MMSE cases, whereas it drops to 2×10^{-8}, 10^{-7}, and 4×10^{-6} with the proposed allocation. As previously, the most significant gain is obtained with the MRC technique because the MAI is canceled when the MC-CDMA system uses the codes provided by our algorithm for $N_u \leq N_c/G$, that is, $G \leq N_c/N_u = 4$ (in simulations, $L \leq 4$ for $N_s \Delta f \sigma_\tau \leq 0.48$) and the performance gain is much lower with the MMSE technique.

When $N_s \Delta f \sigma_\tau$ increases, that is, when the degree of correlation between subcarriers decreases, the performance gain decreases for all detection techniques, because in the extreme case of uncorrelated subcarriers, the BER

Figure 4. BER versus N_u for the different detection techniques with random allocation and with the proposed allocation.
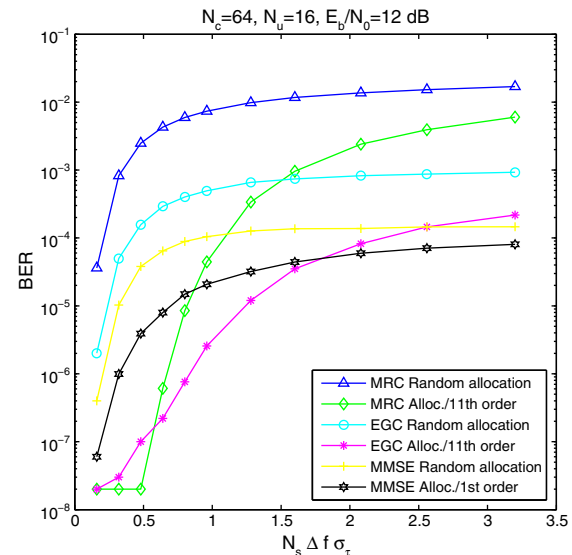

Figure 5. BER versus $N_s \Delta f \sigma_\tau$ for the different detection techniques with random allocation and with the proposed allocation.

no longer depends on the codes assigned to the active users [13].

Finally, the best performance is obtained when the proposed code allocation algorithm is combined with MRC for low values of $N_s \Delta f \sigma_\tau$ (≤ 0.5), with EGC for values of $N_s \Delta f \sigma_\tau$ between 0.5 and 1.7, and with the MMSE technique for a lower correlation between the subcarriers, $N_s \Delta f \sigma_\tau \geq 1.7$. Performance results are given here for Walsh codes, but the algorithm can be applied to other code families. According the expression of the MAI power (8), the benefit of code allocation is even greater that the variation of the code product PCF versus the code couples (z_u, z_v) is large.

5.3. Comparison with multiuser detection

The advantage of our code allocation strategy is to mitigate MAI in downlink MC-CDMA systems without increasing the receiver complexity in the mobile terminal. Another well-known approach much more demanding in computational capacity is MUD, but what about the gain in performance?

To compare both solutions, we consider MUD based on interference cancellation whose aim is to estimate the MAI and to suppress it [2]. More precisely, we have implemented a two-stage parallel interference canceller (PIC), which is more efficient than successive interference canceller for users having the same power [9], as in our simulations. In the first stage, SUD is applied to the received signal to estimate the contribution of all the interferers in the MAI term. It includes channel equalization and despreading by the code of each interfering user. Then, the detected symbols are passed through the channel and subtracted from the received signal. In the second stage, SUD is applied to the resulting signal: equalization by a technique that can be different from the first stage and despreading by the code of the desired user. The first equalization is based on MMSE, which is the best SUD technique in the presence of MAI. For the second stage, MRC, EGC, and MMSE are considered.

Figure 6(a) represents the BER of a MC-CDMA system using SUD combined with our code allocation and MUD with random code allocation versus the load for $N_s \Delta f \sigma_\tau = 0.48$. Our code allocation algorithm uses the 11th-order power approximation, which gives the optimal code sets and leads to the minimum BER. First, we can notice that the three PIC-based MUD techniques provide close performance, slightly better than MMSE SUD with random code allocation if we compare with those in Figure 4. The lower BER is obtained by using MMSE at both stages of the PIC. Now, with our code allocation scheme, the three SUD techniques greatly outperform MUD for $N_u \leq N_c/G = 16$. It is more efficient to avoid MAI by selecting the best code sets than to estimate MAI and to remove it. For a load between N_c/G and $N_c/2 = 32$, EGC-based and MMSE-based SUDs still give better results, but the performance gap is strongly reduced. For a loading rate higher than $1/2$, the results are reversed and the performance obtained with PIC-based MUD is slightly better than with SUD using MMSE and our code allocation.

We can conclude that SUD associated with the proposed code allocation strategy leads to a less-complex receiver than MUD with better performance for $N_u \leq N_c/2$ and almost as good performance for $N_u > N_c/2$ when MMSE is used. For comparison, with MUD, the complexity is approximately multiplied by N_u because the receiver requires the implementation of N_u SUD modules. Moreover additional signaling is needed to send the spreading codes of all the active users to each user receiver.

To go further, it is interesting to try to improve MUD performance by using the code sets given by our code allocation scheme. Figure 6(b) represents the BER obtained by SUD and MUD, both combined with our code allocation. As previously, the 11th-order approximation is used in the algorithm. Whatever the equalizer and the load, the improvement of MUD performance is significant.

More precisely in a low-loaded system, for $N_u \leq N_c/G = 16$, the MAI is completely avoided in the case of PIC MMSE–MRC and PIC MMSE–MMSE, as for MRC-based
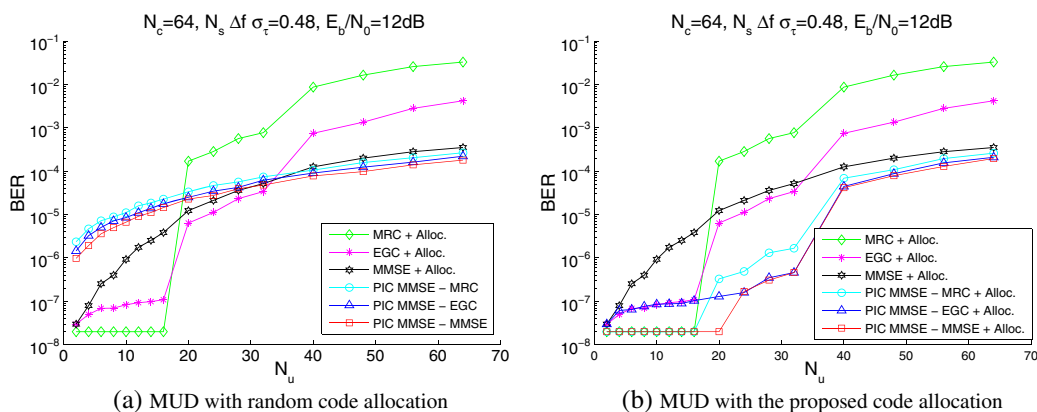

(a) MUD with random code allocation

(b) MUD with the proposed code allocation

Figure 6. Performance comparison between SUD with code allocation and MUD.

SUD, and the performance remains constant, equal to 2×10^{-8}. For a load higher than $N_c/G = 16$ users, MUD outperforms SUD. Thanks to the allocated code sets, the MAI is lower and is more accurately estimated in the first stage of the PIC. Then, after MAI subtraction, the detection results are inherently better in the second stage. For a load higher than $N_c/2 = 32$, the performance difference becomes smaller, but MUD still outperforms MMSE, which is the best SUD technique. Finally, the simulation results show that to significantly improve the performance of a downlink MC-CDMA system, it is worth combining our code allocation algorithm with PIC-based MUD at the expense of a greater computational complexity.

6. CONCLUSION

In this paper, we propose a spreading code allocation strategy for a downlink MC-CDMA system transmitting through a realistic correlated Rayleigh fading channel. The allocation problem is formulated as a minimization problem of a cost function related to MAI. The proposed cost function is based on an approximation of the exact expression of the MAI power for MRC, EGC, and MMSE equalization in the case of a typical exponential PDP without performance loss. The allocation algorithm is iterative and uses the minimum maximum criterion to select the spreading codes of the active users. Unlike previous methods, our algorithm has all the following advantages at once: application to any code family and any equalization technique, low computational cost, and efficient management of load variations.

Via simulation results, we provide a complete analysis of the code allocation efficiency. They show that the algorithm always provides the optimal code sets and consequently leads to the minimum BER when using the suitable approximation order. It turns out that the first order is enough in all cases with MMSE equalization and for a loading rate above 1/4 with MRC and EGC equalization. Nevertheless, for a lower load, it represents a very good compromise between BER and complexity.

The performance gain compared with a random allocation of codes is all the more significant as the faded subcarriers are correlated, that is, as the parameter $N_s \Delta f \sigma_\tau$ is low and as the loading rate is low, in particular, less than 1/2. Moreover it depends on the equalization technique used at the receiver side. The MRC technique, which is the most sensitive to MAI, offers the highest gain, especially in a low-loaded system. Indeed, for $N_u \leq N_c/G$, the MC-CDMA system is MAI free as in the algorithm in [22]. At the opposite, with the MMSE equalizer, which already partially restores code orthogonality, the gain is lower.

At last, the performance of an MC-CDMA system using the proposed allocation strategy can be optimized by introducing equalization control based on the system load and on the subcarrier correlation degree. When the load and the channel are such that $N_u \leq N_c/L$, the MRC technique shall be preferred because it cancels the MAI

and achieves the optimal performance as in a single-user context. For a loading rate more important but less than 1/2 and for a "medium" correlation between the subcarriers $0.5 \leq N_s \Delta f \sigma_\tau \leq 1.7$, the EGC technique provides the best performance. Beyond, the MMSE technique is more effective. Another solution to improve the performance consists in combining our code allocation algorithm with PIC-based MUD, but it is at the expense of a greater computational complexity.

APPENDIX A: FREQUENCY CORRELATION FUNCTION OF THE EQUALIZED CHANNEL $R_{Hg}(n)$

Whatever the equalizer, the coefficient $H_k g_k$ affecting the kth subcarrier depends on the channel envelope $|H_k|$. The correlation function $R_{Hg}(n) = E[H_k g_k H_{k+n}{}^* g_{k+n}{}^*]$ can then be derived from the joint density of $|H_k|$ and $|H_{k+n}|$. For a Rayleigh fading channel, the density is given by [26]

$$p(r_1, r_2) = \frac{4 r_1 r_2}{1 - \rho(n)^2} e^{-\frac{r_1^2 + r_2^2}{1 - \rho(n)^2}} I_0\left(r_1 r_2 \frac{2\rho(n)}{1 - \rho(n)^2}\right)$$
(A.1)

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind and $\rho(n)^2$ is the absolute square of the correlation coefficient between the channel coefficients H_k and H_{k+n}. Because the channel coefficients have zero mean and unitary power, $\rho(n)^2 = |E[H_k H_{k+n}^*]|^2 = |R_H(n)|^2$, where $R_H(n)$ is the FCF before equalization given by (3).

For MRC detection, the channel FCF after equalization is calculated using some results from [32] as

$$R_{Hg}^{\mathrm{MRC}}(n) = E[|H_k|^2 |H_{k+n}|^2]$$

$$= \int_0^\infty \int_0^\infty r_1^2\, r_2^2\, p(r_1, r_2)\, \mathrm{d}r_1\, \mathrm{d}r_2$$

$$= 2\left(1 - \rho(n)^2\right) \int_0^\infty r_1^3 e^{-\frac{r_1^2}{1 - \rho(n)^2}}$$

$$\times {}_1F_1\left(2; 1; r_1^2 \frac{\rho(n)^2}{1 - \rho(n)^2}\right) \mathrm{d}r_1$$

$$= \left(1 - \rho(n)^2\right)^3 F(2, 2; 1; \rho(n)^2)$$

$$= F(-1, -1; 1; \rho(n)^2) = 1 + \rho(n)^2 \quad \text{(A.2)}$$

where ${}_1F_1(\cdot)$ and $F(\cdot)$ are hypergeometric functions [32].

Now, if we consider EGC detection, we get

$$R_{Hg}^{\mathrm{EGC}}(n) = E[|H_k| |H_{k+n}|]$$

$$= \int_0^\infty \int_0^\infty r_1\, r_2\, p(r_1, r_2)\, \mathrm{d}r_1\, \mathrm{d}r_2$$

$$= \sqrt{\pi\left(1 - \rho(n)^2\right)} \int_0^\infty r_1^2 e^{-\frac{r_1^2}{1 - \rho(n)^2}}$$

$$\times {}_1F_1\left(\frac{3}{2}; 1; r_1^2 \frac{\rho(n)^2}{1 - \rho(n)^2}\right) \mathrm{d}r_1$$

$$R_{Hg}^{\text{EGC}}(n) = \frac{\pi}{4}\left(1 - \rho(n)^2\right)^2 F(\frac{3}{2}, \frac{3}{2}; 1; \rho(n)^2)$$

$$= \frac{\pi}{4} F\left(-\frac{1}{2}, -\frac{1}{2}; 1; \rho(n)^2\right) \qquad \text{(A.3)}$$

We can note that for low values of $\rho(n)^2$, the expansion of the hypergeometric function provides the following approximation:

$$R_{Hg}^{\text{EGC}}(n) \approx \frac{\pi}{4}\left(1 + \frac{\rho(n)^2}{4}\right) \qquad \text{(A.4)}$$

For MMSE-based detection, the FCF is written as

$$R_{Hg}^{\text{MMSE}}(n) = E\left[\frac{|H_k|^2}{|H_k|^2 + \sigma^2}\frac{|H_{k+n}|^2}{|H_{k+n}|^2 + \sigma^2}\right]$$

$$= \int_0^\infty \int_0^\infty \frac{r_1^2}{r_1^2 + \sigma^2}\frac{r_2^2}{r_2^2 + \sigma^2}\, p(r_1, r_2)\, \mathrm{d}r_1\, \mathrm{d}r_2$$

$$\text{(A.5)}$$

To calculate this integral, $I_0(\cdot)$ is expressed in terms of a series in the joint density $p(r_1, r_2)$ (A.1). The FCF becomes

$$R_{Hg}^{\text{MMSE}}(n) = \sum_{k=0}^{+\infty} \frac{4\rho(n)^{2k}}{(k!)^2(1 - \rho(n)^2)^{2k+1}}$$

$$\times \left(\int_0^\infty \frac{r_1^{2k+3}}{r_1^2 + \sigma^2} e^{\frac{-r_1^2}{1 - \rho(n)^2}}\, \mathrm{d}r_1\right)^2$$

$$= \sum_{k=0}^{+\infty} \frac{4\rho(n)^{2k}}{(k!)^2(1 - \rho(n)^2)^{2k+1}}$$

$$\times \left(\frac{1}{2}\sigma^{2(k+1)}(k+1)!\right)^2$$

$$\times \left(e^{\frac{\sigma^2}{1 - \rho(n)^2}}\Gamma\left(-(k+1), \frac{\sigma^2}{1 - \rho(n)^2}\right)\right)^2$$

$$= \frac{e^{2\lambda(n)}}{\alpha(n)} \sum_{k=1}^{+\infty} k^2\rho(n)^{2k}\lambda(n)^{2k}\Gamma(-k, \lambda(n))^2$$

$$\text{(A.6)}$$

where $\Gamma(\cdot)$ is the incomplete gamma function, $\alpha(n) = \rho(n)^2/(1 - \rho(n)^2)$ and $\lambda(n) = \sigma^2/(1 - \rho(n)^2)$.

APPENDIX B: SUM OF THE CODE PRODUCT PERIODIC CORRELATION FUNCTION $\sum_{n=1}^{N_c-1} R_\omega^{u,v}(n), u \neq v$

By replacing the PCF of the chip-by-chip product $\omega_k^{u,v} = c_k^{u*}c_k^v$ of the spreading codes of users u and v by (9), we

can rewrite the sum as

$$\sum_{n=1}^{N_c-1} R_\omega^{u,v}(n) = \frac{1}{N_c}\sum_{n=1}^{N_c-1}\sum_{k=0}^{N_c-1}\omega_k^{u,v}\omega_{k+n}^{u,v\,*} \qquad \text{(B.7)}$$

Because the code product is periodic with period N_c, the sum is rewritten as

$$\sum_{n=1}^{N_c-1} R_\omega^{u,v}(n) = \frac{1}{N_c}\sum_{k=0}^{N_c-1}\omega_k^{u,v}\sum_{\substack{n=0\\n\neq k}}^{N_c-1}\omega_n^{u,v*}$$

$$= \frac{1}{N_c}\sum_{k=0}^{N_c-1}\omega_k^{u,v}\sum_{n=0}^{N_c-1}\omega_n^{u,v*}$$

$$- \frac{1}{N_c}\sum_{k=0}^{N_c-1}|\omega_k^{u,v}|^2 \qquad \text{(B.8)}$$

Whatever the code family (Walsh, orthogonal Gold, Golay, or CI), $|\omega_k^{u,v}|^2 = 1$. For binary codes (Walsh, orthogonal Gold, and Golay), the sequence $\{\omega_k^{u,v}, k = 0, \ldots, N_c - 1\}$ contains $N_c/2$ terms equal to -1 and $N_c/2$ terms equal to $+1$, so their sum is zero. In the case of complex CI codes, the sum is also zero:

$$\sum_{k=0}^{N_c-1}\omega_k^{u,v} = \sum_{k=0}^{N_c-1}e^{-jk\frac{2\pi}{N_c}u}e^{jk\frac{2\pi}{N_c}v} = 0 \qquad \text{(B.9)}$$

Finally, we get the following result:

$$\sum_{n=1}^{N_c-1} R_\omega^{u,v}(n) = -1 \qquad \text{(B.10)}$$

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AUTHORS' BIOGRAPHIES

Christelle Garnier received her M.S. and Ph.D. degrees in Signal Processing and Telecommunications from the University of Rennes, France, in 1995 and 1999, respectively. Since 2000, she has been with the Department of Communication Systems at the engineering school TELECOM Lille 1, France, as an assistant professor in mobile communications. Her research interests include digital communications, in particular, multicarrier and spread spectrum transmission schemes, and statistical signal processing, in particular, Monte Carlo methods.

Yves Delignon received his M.S. degree in 1990 from the University of Technology of Compiègne, France. He joined TELECOM Bretagne during the same year for preparing his Ph.D. about statistical modeling of radar images, and he received his Ph.D. degree from the University of Rennes, France, in 1993. In 1992, he joined the engineering school TELECOM Lille 1. From 1993 to 2003, he was an assistant professor and conducted his research in the field of statistical radar image processing. Since 2004, he has been a full professor. He has focused his research on statistical signal processing for digital communications. In

2005, he became Dean of Research at TELECOM Lille 1. His teaching activity mainly concerns statistical signal processing and mobile communication.

Hassan El Ghazi received his M.S. degree in Wireless Communications and his Ph.D. degree in Electrical Engineering from the University of Valenciennes, France, in 2004 and 2008, respectively. From 2007 to 2009, he worked as a research engineer at TeSA, the laboratory of Telecommunications for Space and Aeronautics, Toulouse, France. Since 2010, he has been an assistant professor at the Institut National des Postes et Télécommunications, Rabat, Morocco. His research interests are related to wireless communications, OFDM, and traffic engineering.

François Septier received his engineer degree in Electrical Engineering and Signal Processing in 2004 from TELECOM Lille 1, France, and his M.S. degree in Digital Communications and his Ph.D. degree in Electrical Engineering both from the University of Valenciennes, France, in 2004 and 2008, respectively. From March 2008 to August 2009, he was a research associate in the Signal Processing and Communications Laboratory, Cambridge University Engineering Department, UK. Since August 2009, he has been an assistant professor in TELECOM Lille 1 / LAGIS UMR CNRS 8219, France. His research focuses on Monte Carlo statistical methods and their application in digital communications, multitarget tracking, and source term estimation.